How hard is it to manipulate voting?

Universität Konstanz

Sven Kosub

Theory of Computing
Department of Computer & Information Science

PZ 10** / `toc.uni.kn`

Lecture Series, October 20, 2016

Theory of Computing Group

- complexity theory and algorithms
- logic and discrete mathematics (e.g., game theory)
- communication networks/theory
- distributed computing
- programming/coding social systems

Voting systems

- **alternatives, candidates,** ...
  $X = \{x_1, \ldots, x_m\}$
- **agents, voters,** ...
  $A = \{a_1, \ldots, a_n\}$
- **preferences, utilities,** ...
  bijective function $\pi_i : X \to \{1, \ldots, m\}$ for each $a_i \in A$
  $\pi_i(x) < \pi_i(y)$ means "voter a_i strictly prefers x over y"
- **social choice function, voting rule,** ...
  $F \ : \ \Pi^n \to X \ : \ (\pi_1, \ldots, \pi_n) \mapsto x_j$

Voting systems

- **alternatives, candidates,** ...
  $X = \{x_1, \ldots, x_m\}$
- **agents, voters,** ...
  $A = \{a_1, \ldots, a_n\}$
- **preferences, utilities,** ...
  bijective function $\pi_i : X \to \{1, \ldots, m\}$ for each $a_i \in A$
  $\pi_i(x) < \pi_i(y)$ means "voter a_i strictly prefers x over y"
- **social choice function, voting rule,** ...
  $F \ : \ \Pi^n \to X \ : \ (\pi_1, \ldots, \pi_n) \mapsto x_j$

Voting systems

- **alternatives, candidates,** ...
  $X = \{x_1, \ldots, x_m\}$
- **agents, voters,** ...
  $A = \{a_1, \ldots, a_n\}$
- **preferences, utilities,** ...
  bijective function $\pi_i : X \to \{1, \ldots, m\}$ for each $a_i \in A$
  $\pi_i(x) < \pi_i(y)$ means "voter a_i strictly prefers x over y"
- **social choice function, voting rule,** ...
  $F \ : \ \Pi^n \to X \ : \ (\pi_1, \ldots, \pi_n) \mapsto x_j$

Voting systems

- **alternatives, candidates,** ...
  $X = \{x_1, \ldots, x_m\}$
- **agents, voters,** ...
  $A = \{a_1, \ldots, a_n\}$
- **preferences, utilities,** ...
  bijective function $\pi_i : X \to \{1, \ldots, m\}$ for each $a_i \in A$
  $\pi_i(x) < \pi_i(y)$ means "voter a_i strictly prefers x over y"
- **social choice function, voting rule,** ...
  $F \;:\; \Pi^n \to X \;:\; (\pi_1, \ldots, \pi_n) \mapsto x_j$

Example: 2014 Winter Olympics

119th IOC Session held in Guatemala City, July 4, 2007:

- $X = \{ \text{ Pyeongchang, Sochi } \}$, $\|X\| = 2$
- $A = \{$ Tamás Aján, Syed Shahid Ali, Béatrice Allen, $\ldots \}$, $\|A\| = 95$
- 48 IOC members: $1 = \pi_i(\text{Sochi})$, $2 = \pi_i(\text{Pyeongchang})$
  47 IOC members: $1 = \pi_i(\text{Pyeongchang})$, $2 = \pi_i(\text{Sochi})$
- $F(\pi_1, \ldots, \pi_n) = \arg\max_{x \in X} \|A_x\|$ where $A_x = \{a_i \in A \mid \pi_i(x) = 1\}$,

$F(\pi_1, \ldots, \pi_{95}) = \text{Sochi}$

Voting manipulation

manipulation = strategic voting, i.e., public preference deviates from private preference

119th IOC Session held in Guatemala City, July 4, 2007:

- voter $a_i \in A$ with preference $\pi_i(\text{Pyeongchang}) = 1$ should vote for Pyeongchang (independently from voting behavior of other voters)
- voter $a_i \in A$ with preference $\pi_i(\text{Sochi}) = 1$ should vote for Sochi (independently from voting behavior of other voters)
- given two alternatives, truthful voting is a dominant strategy

Voting manipulation

manipulation = strategic voting, i.e., public preference deviates from private preference

119th IOC Session held in Guatemala City, July 4, 2007:

- voter $a_i \in A$ with preference $\pi_i(\text{Pyeongchang}) = 1$ should vote for Pyeongchang (independently from voting behavior of other voters)
- voter $a_i \in A$ with preference $\pi_i(\text{Sochi}) = 1$ should vote for Sochi (independently from voting behavior of other voters)
- given two alternatives, truthful voting is a dominant strategy

Voting manipulation

manipulation = strategic voting, i.e., public preference deviates from private preference

119th IOC Session held in Guatemala City, July 4, 2007:

- voter $a_i \in A$ with preference $\pi_i(\text{Pyeongchang}) = 1$ should vote for Pyeongchang (independently from voting behavior of other voters)
- voter $a_i \in A$ with preference $\pi_i(\text{Sochi}) = 1$ should vote for Sochi (independently from voting behavior of other voters)
- given two alternatives, truthful voting is a dominant strategy

Voting manipulation

manipulation = strategic voting, i.e., public preference deviates from private preference

119th IOC Session held in Guatemala City, July 4, 2007:

- voter $a_i \in A$ with preference $\pi_i(\text{Pyeongchang}) = 1$ should vote for Pyeongchang (independently from voting behavior of other voters)
- voter $a_i \in A$ with preference $\pi_i(\text{Sochi}) = 1$ should vote for Sochi (independently from voting behavior of other voters)
- given two alternatives, truthful voting is a dominant strategy

Voting manipulation

Gibbard-Satterthwaite Theorem

For $\|X\| \geq 3$, each (surjective) voting rule such that truthful voting is a dominant strategy for each voter is dictatorial.

interpretation:

- impossibility result
- generally, voting manipulation inavoidable

way out:

- voting rules that make it hard to find a manipulation

Voting manipulation

Gibbard-Satterthwaite Theorem

For $\|X\| \geq 3$, each (surjective) voting rule such that truthful voting is a dominant strategy for each voter is dictatorial.

interpretation:

- impossibility result
- generally, voting manipulation inavoidable

way out:

- voting rules that make it hard to find a manipulation

Voting manipulation

Gibbard-Satterthwaite Theorem

For $\|X\| \geq 3$, each (surjective) voting rule such that truthful voting is a dominant strategy for each voter is dictatorial.

interpretation:

- impossibility result
- generally, voting manipulation inavoidable

way out:

- voting rules that make it hard to find a manipulation

Combinatorial voting manipulation

- **constructive manipulation (for voting rule F):**

input: candidate set X;

preference profiles $(\pi_1, \ldots, \pi_k)$ with multiplicities $(w_1, \ldots, w_k)$ for non-manipulators;

multiplicities $(w_{k+1}, \ldots, w_n)$ for manipulators;

candidate $p \in X$

output: preference profiles $(\pi_{k+1}, \ldots, \pi_n)$ such that p wins the voting w.r.t. voting rule F

Combinatorial voting manipulation

- **destructive manipulation (for voting rule F):**

input: candidate set X;

preference profiles $(\pi_1, \ldots, \pi_k)$ with multiplicities $(w_1, \ldots, w_k)$ for non-manipulators;

multiplicities $(w_{k+1}, \ldots, w_n)$ for manipulators;

candidate $p \in X$

output: preference profiles $(\pi_{k+1}, \ldots, \pi_n)$ such that a candidate from $X \setminus \{p\}$ wins the voting w.r.t. voting rule F

Classification of voting rules

- **positional scoring protocols**
- **single transferable vote**
- **method of pairwise comparisons**
- ...

Positional scoring protocols

- **positional scoring protocol (for m candidates):**

$$F(\pi_1, \ldots, \pi_n; w_1, \ldots, w_n) =_{\text{def}} \arg\max_{x \in X} \sum_{a_i \in A} w_i \cdot \alpha_{\pi_i(x)}$$

for scoring vector $\alpha = (\alpha_1, \ldots, \alpha_m), \alpha_j \in \mathbb{Z}, \alpha_1 \geq \cdots \geq \alpha_m$

Example: 2014 Winter Olympics

119th IOC Session held in Guatemala City, July 4, 2007:

- $X = \{$ Pyeongchang, Salzburg, Sochi $\}$, $\|X\| = 3$
- $A = \{$ Tamás Aján, Syed Shahid Ali, Béatrice Allen, $\dots\}$, $\|A\| = 95$

	20	14	11	14	12	4	8	12	95
Pyeongchang	3.	2.	2.	3.	1.	1.	1.	1.	
Salzburg	2.	3.	1.	1.	2.	3.	2.	3.	
Sochi	1.	1.	3.	2.	3.	2.	3.	2.	

Positional scoring protocols

majority: $\alpha = (1, 0, 0)$

	20	14	11	14	12	4	8	12	
Pyeongchang	0	0	0	0	1	1	1	1	36
Salzburg	0	0	1	1	0	0	0	0	25
Sochi	1	1	0	0	0	0	0	0	34

veto: $\alpha = (0, 0, -1)$

	20	14	11	14	12	4	8	12	
Pyeongchang	-1	0	0	-1	0	0	0	0	-34
Salzburg	0	-1	0	0	0	-1	0	-1	-30
Sochi	0	0	-1	0	-1	0	-1	0	-31

Positional scoring protocols

majority: $\alpha = (1, 0, 0)$

	20	14	11	14	12	4	8	12	
Pyeongchang	0	0	0	0	1	1	1	1	36
Salzburg	0	0	1	1	0	0	0	0	25
Sochi	1	1	0	0	0	0	0	0	34

veto: $\alpha = (0, 0, -1)$

	20	14	11	14	12	4	8	12	
Pyeongchang	-1	0	0	-1	0	0	0	0	-34
Salzburg	0	-1	0	0	0	-1	0	-1	-30
Sochi	0	0	-1	0	-1	0	-1	0	-31

Positional scoring protocols: constructive manipulation

majority: $\alpha = (1, 0, 0)$

	20	14	11	14	12	4	8	12
Pyeongchang	3.	2.	2.	3.	1.			
Salzburg	2.	3.	1.	1.	2.			
Sochi	1.	1.	3.	2.	3.			

manipulation goal: Salzburg

	20	14	11	14	12	4	8	12
Pyeongchang	0	0	0	0	1			
Salzburg	0	0	1	1	0			
Sochi	1	1	0	0	0			

Positional scoring protocols: constructive manipulation

majority: $\alpha = (1, 0, 0)$

	20	14	11	14	12	4	8	12
Pyeongchang	3.	2.	2.	3.	1.	2.	2.	2.
Salzburg	2.	3.	1.	1.	2.	1.	1.	1.
Sochi	1.	1.	3.	2.	3.	3.	3.	3.

manipulation goal: Salzburg

	20	14	11	14	12	4	8	12	
Pyeongchang	0	0	0	0	1	0	0	0	12
Salzburg	0	0	1	1	0	1	1	1	49
Sochi	1	1	0	0	0	0	0	0	34

Positional scoring protocols: Borda count

Borda count: $\alpha = (2, 1, 0)$

	20	14	11	14	12	4	8	12
Pyeongchang	3.	2.	2.	3.	1.	1.	1.	1.
Salzburg	2.	3.	1.	1.	2.	3.	2.	3.
Sochi	1.	1.	3.	2.	3.	2.	3.	2.

	20	14	11	14	12	4	8	12	
Pyeongchang	0	1	1	0	2	2	2	2	97
Salzburg	1	0	2	2	1	0	1	0	90
Sochi	2	2	0	1	0	1	0	1	98

Positional scoring protocols: constructive manipulation

Borda count: $\alpha = (2, 1, 0)$

	20	14	11	14	12	4	8	12
Pyeongchang	3.	2.	2.	3.	1.			
Salzburg	2.	3.	1.	1.	2.			
Sochi	1.	1.	3.	2.	3.			

manipulation goal: Pyeongchang

	20	14	11	14	12	4	8	12
Pyeongchang	0	1	1	0	2			
Salzburg	1	0	2	2	1			
Sochi	2	2	0	1	0			

Positional scoring protocols: constructive manipulation

Borda count: $\alpha = (2, 1, 0)$

	20	14	11	14	12	4	8	12	
Pyeongchang	3.	2.	2.	3.	1.	1.	1.	1.	
Salzburg	2.	3.	1.	1.	2.				
Sochi	1.	1.	3.	2.	3.				

manipulation goal: Pyeongchang

	20	14	11	14	12	4	8	12	
Pyeongchang	0	1	1	0	2	2	2	2	97
Salzburg	1	0	2	2	1				82
Sochi	2	2	0	1	0				82

Positional scoring protocols: constructive manipulation

Borda count: $\alpha = (2, 1, 0)$

	20	14	11	14	12	4	8	12
Pyeongchang	3.	2.	2.	3.	1.	1.	1.	1.
Salzburg	2.	3.	1.	1.	2.	2.	2.	3.
Sochi	1.	1.	3.	2.	3.	3.	3.	2.

manipulation goal: Pyeongchang

	20	14	11	14	12	4	8	12	
Pyeongchang	0	1	1	0	2	2	2	2	97
Salzburg	1	0	2	2	1	1	1	0	94
Sochi	2	2	0	1	0	0	0	1	94

Positional scoring protocols: constructive manipulation

Borda count: $\alpha = (2, 1, 0)$

	20	14	11	14	12	8	8	8	
Pyeongchang	3.	2.	2.	3.	1.	1.	1.	1.	
Salzburg	2.	3.	1.	1.	2.	2.	2.	3.	
Sochi	1.	1.	3.	2.	3.	3.	3.	2.	

manipulation goal: Pyeongchang

	20	14	11	14	12	8	8	8	
Pyeongchang	0	1	1	0	2	2	2	2	97
Salzburg	1	0	2	2	1	1	1	0	98
Sochi	2	2	0	1	0	0	0	1	90

Positional scoring protocols

Dichotomy theorem für positional scoring protocols
[Conitzer, Sandholm, Lang 2007; Hemaspaandra, Hemaspaandra 2007]

For a positional scoring protocol with scoring vector $\alpha = (\alpha_1, \ldots, \alpha_m)$*, the constructive manipulation problem*

- *can be solved in polynomial time, if* $\alpha_2 = \cdots = \alpha_m$*,*
- *is* NP*-complete, otherwise.*

(The statement is true already for three candidates.)

Consequence:

- majority easy to manipulate constructively
- veto and Borda count hard to manipulate constructively

Positional scoring protocols

Dichotomy theorem für positional scoring protocols
[Conitzer, Sandholm, Lang 2007; Hemaspaandra, Hemaspaandra 2007]

For a positional scoring protocol with scoring vector $\alpha = (\alpha_1, \ldots, \alpha_m)$, *the constructive manipulation problem*

- *can be solved in polynomial time, if* $\alpha_2 = \cdots = \alpha_m$,
- *is* NP-*complete, otherwise.*

(The statement is true already for three candidates.)

Consequence:

- majority easy to manipulate constructively
- veto and Borda count hard to manipulate constructively

Vote transfers

- **single transferable vote (for m candidates):**

  votes are collected in $m - 1$ rounds;

  in each round, the candidate with least number of votes (w.r.t. majority) is eliminated;

  eliminated candidates are deleted from all preference profiles (i.e., votes are transferred to next ranks);

  in final round, winner is determined w.r.t. majority

Vote transfers

first round

	20	14	11	14	12	4	8	12	
Pyeongchang	3.	2.	2.	3.	1.	1.	1.	1.	
Salzburg	2.	3.	1.	1.	2.	3.	2.	3.	
Sochi	1.	1.	3.	2.	3.	2.	3.	2.	

majority: $\alpha = (1, 0, 0)$

	20	14	11	14	12	4	8	12	
Pyeongchang	0	0	0	0	1	1	1	1	36
Salzburg	0	0	1	1	0	0	0	0	25
Sochi	1	1	0	0	0	0	0	0	34

Vote transfers

second round

	20	14	11	14	12	4	8	12	
Pyeongchang	2.	2.	1.	2.	1.	1.	1.	1.	
Salzburg	–	–	–	–	–	–	–	–	
Sochi	1.	1.	2.	1.	2.	2.	2.	2.	

majority: $\alpha = (1, 0)$

	20	14	11	14	12	4	8	12	
Pyeongchang	0	0	1	0	1	1	1	1	47
Salzburg	–	–	–	–	–	–	–	–	–
Sochi	1	1	0	1	0	0	0	0	48

Complexity of voting manipulation

	constructive				destructive	
voting rule	**2**	**3**	**$4, 5, 6$**	**≥ 7**	**2**	**≥ 3**
majority	P	P	P	P	P	P
veto	P	NP	NP	NP	P	P
Borda	P	NP	NP	NP	P	P
single transferable vote	P	NP	NP	NP	P	NP
majority with run-off	P	NP	NP	NP	P	NP
regular cup	P	P	P	P	P	P
Copeland	P	P	NP	NP	P	P
Simpson (Maximin)	P	P	NP	NP	P	P
Schulze	P	P	P	P	P	P

Example: 2014 Winter Olympics

119th IOC Session held in Guatemala City, July 4, 2007: What really happened?

- $X = \{\ \text{Pyeongchang}, \text{Salzburg}, \text{Sochi}\ \}$, $\|X\| = 3$
- overall 111 IOC members but 13 excluded from voting, i.e., $\|A\| = 98$
- voting rule: single transferable vote

candidate	**1st round**	**2nd round**
Sochi	34	51
Pyeongchang	36	47
Salzburg	25	–

(3 IOC members from AUT/GER entitled to vote in 2nd round)

Regular lectures etc.

Theory

- Complexity Theory (WS 16, SS 18, 4+2) $\longrightarrow$ starts Tue, Oct 25, 5pm
- Logic in Computer Science (SS 17, 4+0/2)
- Design and Analysis of Algorithms (WS, 4+2)
- Graph Drawing (SS, 4+2)
- ...
- individual bachelor's/master's projects and theses $\longrightarrow$ by appointment

Compatible study profiles: everything related to algorithmics!

- Network Science
- Data Mining/Big Data
- Systems

Regular lectures etc.

Theory

- Complexity Theory (WS 16, SS 18, 4+2) $\longrightarrow$ starts Tue, Oct 25, 5pm
- Logic in Computer Science (SS 17, 4+0/2)
- Design and Analysis of Algorithms (WS, 4+2)
- Graph Drawing (SS, 4+2)
- ...
- individual bachelor's/master's projects and theses $\longrightarrow$ by appointment

Compatible study profiles: everything related to algorithmics!

- Network Science
- Data Mining/Big Data
- Systems