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Network Dynamics
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# Assignment 12

**Ausgabe:** 22 Jan 2014 **Abgabe:** 29 Jan 2014

## Problem 1: Ordinal potential games — 10 Points

Prove or disprove that the following bimatrix game

$$\Gamma = \begin{pmatrix} (1,2) & (2,3) & (3,1) \\ (2,2) & (3,1) & (1,3) \\ (3,3) & (1,1) & (2,2) \end{pmatrix}$$

is an ordinal potential game.

*Hint*: Check the Finite Improvement Property for $\Gamma$.

## Problem 2: Improvement paths — 10 Points

Find a nondegenerate $2 \times 3$-game $\Gamma$ (i.e., a game with 2 players each of which has 3 strategies) such that the length of the longest (finite) improvement path of $\Gamma$ is minimum among all nondegenerate $2 \times 3$-games.

*Hint*: Construct an appropriate strict order for $\Gamma$.

## Problem 3: Netlogo — 10 Points

For a loopless, directed graph $G = (V, E)$ with $n$ vertices, the *in-degree distribution* $\mathbf{P}^{\text{in}} : \{0, \ldots, n-1\} \to [0, 1]$ is given by

$$\mathbf{P}^{\text{in}}(i) = \frac{\|\{\ v \in V \mid \text{in-deg}_G(v) = i\ \}\|}{n}.$$

Identify $\mathbf{P}^{\text{in}}$ with a sequence $p = (p_0, p_1, \ldots, p_{n-1})$ such that $p_i = \mathbf{P}^{\text{in}}(i)$, and define the *entropy* $H(p)$ of the sequence $p$ by

$$H(p) =_{\text{def}} -\sum_{i=0}^{n-1} p_i \log_2(p_i),$$

where $\log_2(0) = 0$ by convention.

Suppose you are given $n$ persons each of which points to exactly one other person. Thus, a loopless, directed graph is constructed. Consider the associated potential game $\Gamma$ (the strategies being pointers to other persons) with the potential function $H$.

(a) Design a Netlogo program for the game $\Gamma$ which implements the best-response principle; ties are broken randomly. Initially, persons point to teach other in a circular fashion, i.e., person 1 points to person 2, person 2 points to person 3, ..., person $n$ points to person 1. Find the maximum entropy of the game $\Gamma$. How does your fixed-point distribution look like?

(b) Run a similar Netlogo program for the game $\Gamma'$ where each person points to exactly two different persons. Assume that in each update step only one pointer can be redirected. Find the maximum entropy of the game $\Gamma'$. How does your fixed-point distribution look like?