University of Konstanz
Department of Computer & Information Science
PD Dr. Sven Kosub / Mehwish Nasim

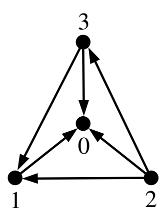
Network Dynamics
Winter 2013/14

Assignment 3

Ausgabe: 6 Nov 2013 **Abgabe:** 13 Nov 2013

Problem 1: Fixed-point analysis — 10 Points

Consider the following AS graph together with total orderings of all path sets:



$$
\begin{aligned}
P^0 &: (0) \succ_0 \varepsilon \\
P^1 &: (1,0) \succ_1 (1,2,0) \succ_1 (1,3,0) \succ_1 (1,3,2,0) =_1 (1,2,3,0) \succ_1 \varepsilon \\
P^2 &: (2,3,0) \succ_2 (2,1,0) \succ_2 (2,1,3,0) \succ_2 (2,3,1,0) \succ_2 (2,0) \succ_2 \varepsilon \\
P^3 &: (3,1,0) \succ_3 (3,1,2,0) \succ_3 (3,0) \succ_3 (3,2,0) \succ_3 (3,2,1,0) \succ_3 \varepsilon
\end{aligned}
$$

The AS graph depicted is oriented according to the customers and providers for each node. Note that a directed edge $u \to v$ indicates that u is a customer of v, and a directed edge $u \leftarrow v$ indicates that u is a provider of v.

(a) Eliminate all paths from the paths sets that are not valley-free or empty.

(b) Determine all customer paths in the remaining path sets.

(c) Is the Gao-Rexford convergence criterion satisfied for the remaining BGP system?

Problem 2: Valley-free orientations — 10 Points

Let $G = (V, E)$ be an undirected graph with vertex set $V = \{0, 1, 2, \ldots, n-1\}$. An orientation of an edge $e = \{u, v\}$, $u < v$, in G replaces e by exactly one of the directed edges $u \to v$ or $u \leftarrow v$. An orientation of G consists of orientations of all edges in G.

(a) Consider the following path set

$$P = \{\ (1,2,5), (1,0,3,4,7), (7,4,5,8), (4,3,6,7)\ \}$$

Find an orientation of the induced AS graph $G[P]$ such that all paths are valley-free and the oriented AS graph is acyclic.

(b) Find a path set P such that an orientation of $G[P]$ cannot satisfy the following both conditions simultaneously:

i. The oriented AS graph $G[P]$ is acyclic.

ii. All paths in P are valley-free.

Problem 3: Valley-free orientations **10 Points**

For a path $p = (v_0, v_1, v_2, \dots, v_m)$ of length $\ell(p) = m \geq 2$, define the set

$$I_p =_{\text{def}} \{ \ (v_{i-1}, v_i, v_{i+1}) \mid i \in \{1, \dots, m-1\} \ \}$$

of paths of length exactly 2. We extend this definition to path sets P by defining

$$I_P =_{\text{def}} \bigcup_{\substack{p \in P \\ \ell(p) \geq 2}} I_p \quad \cup \quad \{ \ p \in P \mid \ell(p) \leq 1 \ \}$$

(a) Determine I_P for the path set $P = \{(1,3,2,5,0), (4,3,2,1), (1,0), (0), (3,5,2)\}$.

(b) Show that for each path set P (not containing the empy path) the following two statements are equivalent:

i. There is an orientation of $G[P]$ such that all paths in P are valley-free.

ii. There is an orientation of $G[I_P]$ such that all paths in I_P are valley-free.