University of Konstanz
Department of Computer & Information Science
PD Dr. Sven Kosub / Mehwish Nasim

Network Dynamics
Winter 2013/14

Assignment 5

Ausgabe: 20 Nov 2013 **Abgabe:** 27 Nov 2013

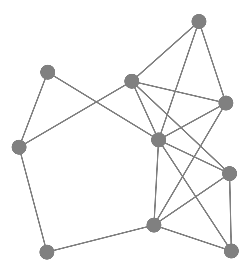
Problem 1: Two-mode networks — 10 Points

Suppose that $X \in \mathbb{R}^{n \times m}$ is a $n \times m$-matrix associated with a two-mode network attribute for A and S such that $\|A\| = n$ and $\|S\| = m$.

Prove that the matrix $X \cdot X^T$ is symmetrical, i.e., it holds that $X \cdot X^T = \left(X \cdot X^T\right)^T$.

Problem 2: Two-mode networks — 10 Points

The *Latent Affiliation Problem* (LAP) for single-attribute networks is the following: Given a single-attribute network x on an interaction domain $\mathcal{I} \subseteq A \times A$, find a two-mode network x' on an affiliation domain $\mathcal{A} \subseteq A \times S$ such that x is a one-mode projection of x'. Consider the following undirected graph $G = (A, E)$:



Solve the LAP for the single-attribute network associated with G.

Problem 3: Iterated maps — 10 Punkte

Find a map $F : D^n \rightarrow D^n$ for each of the sequences $(x_0, x_1, \dots, x_n, \dots)$ listed below such that the given sequence is the trajectory of x_0 with respect to F, if such that an F exists; otherwise prove that no such F exists.

(a) $(0, 1, \sqrt{2}, \sqrt{3}, \dots, \sqrt{n}, \dots)$ for $D = \mathbb{R}_{\geq 0}$, $n = 1$

(b) $(1, 1/4, 1/9, 1/16, \dots, (n+1)^{-2}, \dots)$ for $D = \mathbb{R}_{\geq 0}$, $n = 1$

(c) $(0, 1, 1, 2, 2, 2, 3, 3, 3, 3, \dots, \underbrace{n, \dots, n}_{n+1 \text{ times}}, \dots)$ for $D = \mathbb{R}_{\geq 0}$, $n = 1$

(d) $(0.a_1a_2a_3\dots,\ 0.a_1a_1a_2a_3\dots,\ 0.a_1a_1a_1a_2a_3\dots,\ \dots,\ 0.\underbrace{a_1\dots a_1}_{n \text{ times}}a_2a_3\dots,\ \dots)$, where $a_i \in \{0, 1\}$ for all $i \in \mathbb{N}$, for $D = [0, 1)$, $n = 1$.

Hint: Consult the example describing the BERNOULLI shift in the lecture notes.

(e) $((0, 1), (1, 1), (1, 2), (2, 3), (3, 5), (5, 8), \dots, (F_n, F_{n+1}), \dots)$ for $D = \mathbb{N}$, $n = 2$, where F_n is the n-the FIBONACCI number, i.e., $F_0 =_{\text{def}} 0$, and $F_n =_{\text{def}} F_{n-1} + F_{n-2}$ for all $n > 0$.