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Network Dynamics
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# Assignment 6

**Ausgabe:** 27 Nov 2013 **Abgabe:** 04 Dec 2013

## Problem 1: Orbits — 10 Points

Let $A \in \mathbb{R}^{2\times 2}$ be a matrix of the following type:

$$A =_{\text{def}} \begin{pmatrix} a & b \\ 0 & a \end{pmatrix}, \qquad a, b \in \mathbb{R}$$

Consider the function $F : \mathbb{R}^2 \to \mathbb{R}^2 : x \mapsto A \cdot x$ (which depends on concrete choices of $a, b$).

Find all pairs $(a, b)$ of values $a, b \in \mathbb{R}$ such that the orbits of $(-1, 1)$ and $(1, 1)$ under $F$ are disjoint.

## Problem 2: Fixed points — 10 Points

Suppose we are given the (symmetrical) interaction domain $\mathcal{I} = \{ \{i, j\} \mid i, j \in A \}$ for a set $A = \{1, \ldots, n\}$. An *open triangle* of the (undirected) network $x : \mathcal{I} \to \{0, 1\}$ is formed by three dyads $\{i, j\}$, $\{j, k\}$, and $\{i, k\}$, where $i, j, k$ are pairwise different, such that $x_{\{i,j\}} = x_{\{j,k\}} = 1$ and $x_{\{i,k\}} = 0$.

Define $F : \{0, 1\}^{\mathcal{I}} \to \{0, 1\}^{\mathcal{I}}$ to be the map that closes all open triangles of a given network $x$, i.e., if dyads $\{i, j\}$, $\{j, k\}$, and $\{i, k\}$ form an open triangle in the network $x$ then in network $F(x)$, it holds that $F(x)_{\{i,j\}} = F(x)_{\{j,k\}} = F(x)_{\{i,k\}} = 1$.

How many fixed points of $F$ (depending on $n$) are associated with connected graphs? Prove your hypothesis.

## Problem 3: Periodic states — 10 Punkte

Consider the Bernoulli shift $F : [0, 1) \to [0, 1) : x \mapsto 2x \mod 1$.

(a) Show that, for each $x \in [0, 1)$, if $x$ is periodic under $F$ then $x$ is rational.

(b) Is it true that, for each $x \in [0, 1)$, if $x$ is rational then $x$ is periodic under $F$?