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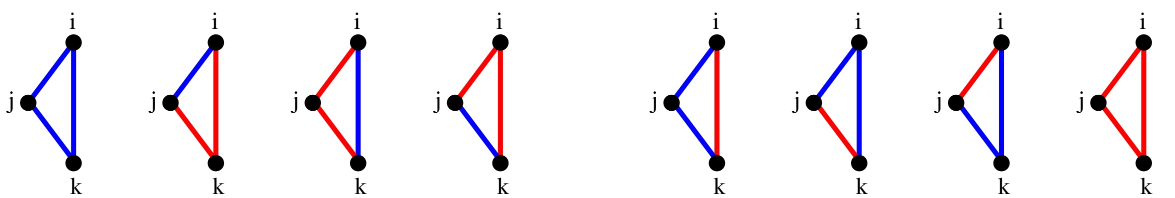
Network Dynamics
Winter 2013/14

Assignment 7

Ausgabe: 04 Dec 2013 **Abgabe:** 11 Dec 2013

Let $\mathcal{I} \subseteq A \times A$ be an arbitrary symmetrical interaction domain for a finite set $A = \{1, \ldots, n\}$ of actors. We consider a categorical network attribute $x : \mathcal{I} \to \{\mathsf{friend}, \mathsf{enemy}\}$. Suppose we are given a triple (i, j, k) of actors $i, j, k \in A$ such that $\{i, j\}, \{j, k\}, \{i, k\} \in \mathcal{I}$. Notice that the order of the actors in the triple is important in the forthcoming.

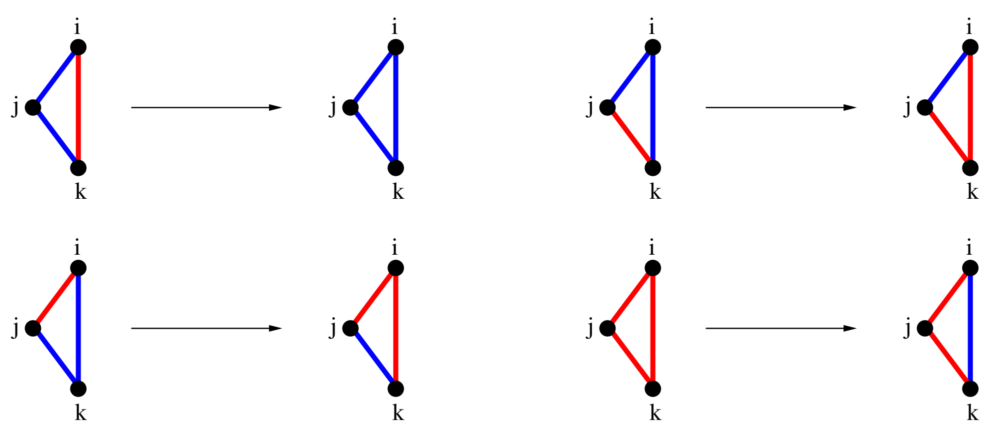
The network attribute restricted to the triple (i, j, k) has one of the following representations as a graph, where a blue edge represents attribute value friend and a red edge represents attribute value enemy.



A friend of my friend is my friend	An enemy of my friend is my enemy	An enemy of my enemy is my friend	A friend of my enemy is my enemy	A friend of my friend is my enemy	An enemy of my friend is my friend	A friend of my enemy is my friend	An enemy of my enemy is my enemy

Note that, when interpreting the graphs, according to the triple (i, j, k), actor i is the focal actor and actor j is closer to actor i than actor k.

The left group of four graphs represents stable situations whereas the right group of four graphs represents instable situations. In these situations, instability is resolved according to the following replacement rules:



Now, suppose a concrete interaction domain over a set $A = \{1, 2, 3, 4\}$ of actors is given:

$$\mathcal{I} =_{\text{def}} \{ \{1, 2\}, \{2, 3\}, \{3, 4\}, \{4, 1\}, \{2, 4\} \}$$

$\mathcal{I}$ consists of the two triangles $\{1, 2, 4\}$ and $\{2, 3, 4\}$.

We consider a total mapping $F : \{\mathsf{friend}, \mathsf{enemy}\}^{\mathcal{I}} \to \{\mathsf{friend}, \mathsf{enemy}\}^{\mathcal{I}}$ which assigns to a network $x : \mathcal{I} \to \{\mathsf{friend}, \mathsf{enemy}\}$ a network $F(x)$ where stable triangles of x are maintained and instable triangles of x are replaced according to the replacement rules. Here, the triangles are understood as triples $(2, 1, 4)$ and $(2, 3, 4)$. In the case that replacement rules for an edge are inconsistent, value enemy beats value friend.

Problem 1: Phase space **30 Points**

(a) Draw the state graph $\Gamma(F)$ of F.

(b) Find the visiting probability of each state in a random orbit of F.