University of Konstanz
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Network Dynamics
Winter 2013/14

Assignment 8

Ausgabe: 11 Dec 2013 **Abgabe:** 18 Dec 2013

Problem 1: Series — 10 Points

Consider the following boolean map

$$F : \{0,1\}^3 \to \{0,1\}^3 : (x_1, x_2, x_3) \mapsto (x_1 \oplus (x_2 \vee x_3), 1 \oplus x_1 \oplus x_2, x_2 \oplus x_3)$$

and the function $\tau : \{0,1\}^3 \to \mathbb{R} : (x_1, x_2, x_3) \mapsto |(x_1, x_2, x_3)|_1 - |(x_1, x_2, x_3)|_0$.

Find the probability that the time series associated with τ and a random orbit of F has a fixed point.

Problem 2: Levels — 10 Points

Let $F : D^n \to D^n$ be a map. Prove or disprove the following statements:

(a) If L_1 and L_2 are levels of F then $L_1 \cap L_2$ is a level of F.

(b) If L_1 and L_2 are levels of F then $L_1 \cup L_2$ is a level of F.

Problem 3: Plots — 10 Points

Consider again the boolean map (from the first problem)

$$F : \{0,1\}^3 \to \{0,1\}^3 : (x_1, x_2, x_3) \mapsto (x_1 \oplus (x_2 \vee x_3), 1 \oplus x_1 \oplus x_2, x_2 \oplus x_3).$$

Determine the Derrida relation $\mathcal{D}(F)$ of F (as a multiset) and represent $\mathcal{D}(F)$ as a matrix $P = (p_{i,j})_{i,j \in \{0,1,2,3\}} \in \mathbb{N}^{4\times 4}$ such that

$$p_{i,j} = \|\{\ (x,y) \mid x, y \in \{0,1\}^3,\ d_H(x,y) = i, \text{ and } d_H(F(x), F(y)) = j\ \}\|.$$

Hint: You may run a computer program to compute the entries of the matrix.