## Assignment 9

## Ausgabe: 18 Dec 2013 Abgabe: 8 Jan 2014

For a map $F:\{0,1\}^{n} \rightarrow\{0,1\}^{n}$, let the modified version of the Derrida relation be defined as follows:

$$
\begin{array}{r}
\mathcal{D}^{\prime}(F)=_{\text {def }}\left\{\left(h_{1}, h_{2}\right) \mid \quad h_{1}<n \text { and there are } x_{1}, x_{2} \text { such that } d_{H}\left(x_{1}, x_{2}\right)=h_{1}\right. \\
\text { and } \left.d_{H}\left(F\left(x_{1}\right), F\left(x_{2}\right)\right)=h_{2}\right\}
\end{array}
$$

Note that the Hamming distance $d_{H}(x, y)$ can be as high as $n$, e.g., for $x=0^{n}$ and $y=1^{n}$. Moreover, define the modified Derrida coefficient $\mathrm{Dc}^{\prime}(F)$ as

$$
\operatorname{Dc}^{\prime}(F)={ }_{\text {def }} \log _{2} \beta,
$$

where $\beta$ is the regression coefficient of the multiset of points of $\mathcal{D}^{\prime}(F)$.

## Problem 1: Plots

10 Points
Determine the value $\mathrm{Dc}^{\prime}(F)$ for the map

$$
F:\{0,1\}^{3} \rightarrow\{0,1\}^{3}:\left(x_{1}, x_{2}, x_{3}\right) \mapsto\left(x_{1} \oplus\left(x_{2} \vee x_{3}\right), 1 \oplus x_{1} \oplus x_{2}, x_{2} \oplus x_{3}\right)
$$

Hint: You may run an appropriate computer program.

## Problem 2: Plots

20 Points
Find a map $F:\{0,1\}^{3} \rightarrow\{0,1\}^{3}$ that maximizes the modified Derrida coefficient, i.e., $F$ should satisfy

$$
\operatorname{Dc}^{\prime}(F) \geq \mathrm{Dc}^{\prime}\left(F^{\prime}\right) \text { for all functions } F^{\prime}:\{0,1\}^{3} \rightarrow\{0,1\}^{3} .
$$

Determine also the maximal value of that map.
Hint: You are strongly encouraged to design and run an appropriate computer program.

## Problem 3: Netlogo

0 Points
Download and install a local copy of the Netlogo software from

```
http://ccl.northwestern.edu/netlogo/
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on your computer.

