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Complexity Theory
Winter 2016

Assignment 11

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Exercise 1.

Consider the 2-person game Node Kayles which is defined on an undirected graph as follows: Player 1 and player 2 alternatingly label one node each in the graph per round. Only those nodes can be labeled that are not yet labeled or are not adjacent to any labeled node. The first player who is not able to label a node anymore loses the game.

A player has a *sure winning strategy* if there is always a move so that independent of the choices of the other player, the player wins the game eventually.

Design an algorithm in alternating pseudocode which decides the following problem in polynomial time:

Input:	undirected graph $G = (\{1, 2, \ldots, n\}, E)$
Question:	Is there a sure winning strategy for player 1 for Node Kayles on G, if player 1 begins?

Hint: You may use set variables and simple operations on sets, like $\in, \subseteq, \setminus, \cap, \cup$.

Exercise 2.

Design an algorithm in alternating pseudocode which decides the set

$$\mathrm{SAT}_\omega =_{\mathrm{def}} \{ H \mid H = H(x_1, \ldots, x_n) \text{ is a propositional formula and } (\exists a_1)(\forall a_2)(\exists a_3) \ldots (Qa_n)[H(a_1, a_2, a_3, \ldots, a_n)] \text{ is true } \}$$

in polynomial time.

Exercise 3.

A tuple (V, E, v_a, v_e) is called *reachability system* if $E \subseteq V^3$ and $v_a, v_e \in V$. Reachability is defined inductively:

- *basis of induction*: v_a is reachable,

- *inductive step*: if u and v are reachable and $(u, v, w) \in E$ then w is reachable.

Design an algorithm in alternating pseudocode which decides the following problem in polynomial time:

Input:	reachability system $(\{1, 2, \dots, n\}, E, v_a, v_e)$
Question:	Is v_e reachable?