University of Konstanz
Department of Computer & Information Science
Prof. Dr. Sven Kosub / Michael Aichem

Complexity Theory
Winter 2016

(Last) Assignment 13

Issue date: 01 Feb 2017 **Due date:** 08 Feb 2017

Exercise 1.

Prove the following closure properties for the class GapP:

(a) If $f, g \in \mathrm{GapP}$ then $f + g \in \mathrm{GapP}$.

(b) If $f, g \in \mathrm{GapP}$ then $f \cdot g \in \mathrm{GapP}$.

(c) If $f, g \in \mathrm{GapP}$ then $f - g \in \mathrm{GapP}$.

(d) If $f \in \mathrm{GapP}$, $g \in \mathrm{FP}$ then $\sum(f, g) \in \mathrm{GapP}$.

(e) If $f \in \mathrm{GapP}$, $g \in \mathrm{FP}$ such that $g(x) \leq p(|x|)$ for some polynomial p then $\Pi(f, g) \in \mathrm{GapP}$.

Exercise 2.

For functions $f, g : \Sigma^* \to \mathbb{N}$, the *modified subtraction* $\ominus$ is defined as

$$f \ominus g : x \mapsto \max\{0, f(x) - g(x)\}.$$

Prove that if $f \ominus g \in \#\mathrm{P}$ for all functions $f \in \#\mathrm{P}$, $g \in \mathrm{FP}$ then $\mathrm{NP} = \mathrm{coNP}$.

Exercise 3.

Prove the following equivalence:

$$\mathrm{PP} = \mathrm{P} \iff \#\mathrm{P} = \mathrm{FP}$$