University of Konstanz
Department of Computer & Information Science
PD Dr. Sven Kosub / David Schoch

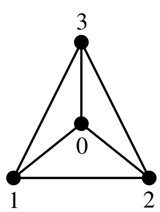
Network Dynamics
Winter 2014/15

Assignment 1

Ausgabe: 22 Oct 2014 **Abgabe:** 29 Oct 2014

Problem 1: Policy routing — 10 Points

Consider the following graph $G = (V, E)$ together with total orderings of all path sets:



$$\begin{aligned}
P^0 &: (0)\\
P^1 &: (1,0) \succ_1 (1,2,0) \succ_1 (1,3,0) \succ_1 (1,3,2,0) =_1 (1,2,3,0) \succ_1 (1)\\
P^2 &: (2,3,0) \succ_2 (2,1,0) \succ_2 (2,1,3,0) \succ_2 (2,3,1,0) \succ_2 (2,0) \succ_2 (2)\\
P^3 &: (3,1,0) \succ_3 (3,1,2,0) \succ_3 (3,0) \succ_3 (3,2,0) \succ_3 (3,2,1,0) \succ_3 (3)
\end{aligned}$$

One stably confluent configuration $\pi : V \to V : u \mapsto \pi(u)$ is

$$\pi(0) = 0, \quad \pi(1) = 0, \quad \pi(2) = 1, \quad \pi(3) = 1$$

(a) Find another stably confluent configuration, if there is any.

(b) How many stably confluent configurations exist in total?

Problem 2: Policy routing — 10 Points

Suppose the graph $G = (V, E)$ your are given is the complete graph K^3 on three vertices, i.e., G consists of vertex set $V = \{0, 1, 2\}$ and edge set $E = \{ \{0, 1\}, \{0, 2\}, \{1, 2\} \}$.

(a) Is there a total ordering of all path sets P^0, P^1, and P^2 such that there is no stably confluent configuration?

(b) How many total orderings of the path sets exist such that there is no stably confluent configuration?

Problem 3: Routing hierarchy — 10 Points

An important aspect of Internet inter-domain routing is the reduction of the number of possible paths that can be selected for routing. For that, we make a distinction between the

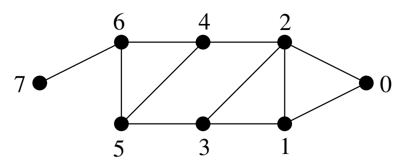
Internet at the *router level* and the Internet at the *AS level*, where AS is an abbreviation for *Autonomous Systems*, i.e., subnets within the Internet.

More formally, suppose we are given an undirected graph $G = (V, E)$ (which represents the Internet at the router level). Let $\mathcal{X} = \{X_1, \ldots, X_r\}$ be any partition of the vertex set V. Then, the *AS graph* $H(G, \mathcal{X})$ is defined to consist of vertex set $\mathcal{X}$ and the edge set

$$\mathcal{A} =_{\text{def}} \{ \{X_i, X_j\} \mid \text{there exist } x \in X_i \text{ and } y \in X_j \text{ such that } \{x, y\} \in E \}.$$

We say that a path $p = (u_0, u_1, \ldots, u_k)$ in G is a *valid AS path* (with respect to $H(G, \mathcal{X})$) if and only if the sequence $(U_0, U_1, \ldots, U_k)$ of sets $U_i \in \mathcal{X}$ such that $u_i \in U_i$ for all $i \in \{0, 1, \ldots, k\}$ satisfies the following property for all $i < \ell < j$: if $U_i = U_j$ then $U_i = U_\ell$.

Consider the following undirected graph $G = (V, E)$ which represent a possible, small portion of the Internet (at the router level):



Furthermore, the following partition of the vertex set $\{0, 1, \ldots, 7\}$ of the graph G is given:

$$A = \{6, 7\}, \quad B = \{2, 4\}, \quad C = \{3, 5\}, \quad D = \{1, 0\}$$

The sets A, B, C, and D are Autonomous Systems.

(a) Determine the number of $(7, 0)$-paths in G.

(b) Determine the AS graph given by the partition of the vertex set of G above.

(c) Determine the number of valid AS paths between nodes 7 and 0.

(d) Which partition of the vertex set V gives the maximum number of valid AS paths?

(e) Which partition of the vertex set V gives the minimum number of valid AS paths?