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Network Dynamics
Winter 2014/15

Assignment 10

Ausgabe: 14 Jan 2015 **Abgabe:** 21 Jan 2015

Problem 1: Potential games — 10 Points

Prove or disprove that the following bimatrix game

$$\Gamma_2 = \begin{pmatrix} (1,0) & (2,0) \\ (2,0) & (0,1) \end{pmatrix}$$

is an ordinal potential game.

Problem 2: Friendship networks — 10 Points

Consider the formation of a friendship network of n neurotic persons. A neurotic person wants to have many friends but wants these friends not to be friends among each other. We formulate this scenario as a strategic game $\Gamma = (A, S, u)$ such that

- $A = \{1, \ldots, n\}$ is the set of persons,
- $S = S_1 \times \cdots \times S_n$ where $S_i = \mathcal{P}(\{\ (i,j) \mid j \in A \setminus \{i\}\ \})$, i.e., i's strategy is basically a set of selected persons; here, we consider friendship as a directed relationship which needs not necessarily be mutually confirmed,
- $u = (u_1, \ldots, u_n)$ where $u_i(s_1, \ldots, s_n)$ is the number of pairs $\{j,k\}$ such that $(i,j), (i,k) \in s_i$ but neither $(j,k) \in s_j$ nor $(k,j) \in s_k$.

Find a Nash equilibrium of Γ for n persons.

Hint: The Nash equilibrium is not unique.

Problem 3: Friendship networks — 10 Points

Consider again the neurotic-network formation process in Problem 2. Assume that the utility functions are modified. That is, we consider a game $\Gamma = (A, S, u')$ where A and S are the same as above but u'_i is defined as

$$u'_i(s_1, \ldots, s_n) =_{\text{def}} \|s_i\| - \|\{\ \{j,k\} \mid (i,j),(i,k) \in s_i \wedge ((j,k) \in s_j \vee (k,j) \in s_k)\ \}\|,$$

i.e., i's utility is the out-degree minus the number of pairs of simply connected friends.

Find a Nash equilibrium of Γ for n persons.

Hint: The Nash equilibrium is not unique.