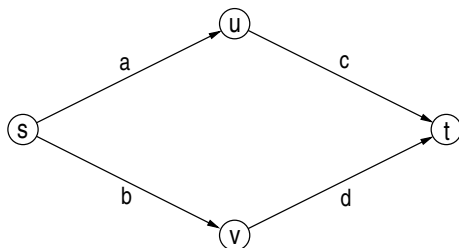# Assignment 12

Ausgabe: 28 Jan 2015   **Abgabe:** 4 Feb 2015

Problem 1: Congestion games   10 Points

Consider a congestion model $(A, F, (S_i)_{i \in A}, (w_f)_{f \in F})$ associated with the following traffic scenario:

where

- the set A of agents is $A = \{1, 2, \ldots, 10\}$,

- the set F of facilities (roads) is $F = \{a, b, c, d\}$,

- the sets $S_i = S$ of strategies (pathways) is the same for all agents: $S = \{ \{a, c\}, \{b, d\} \}$

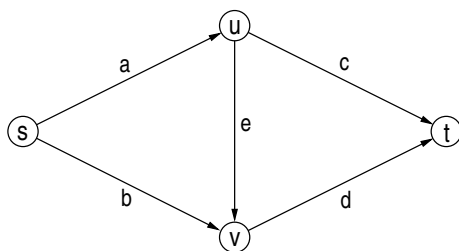- the cost functions w_f for each facility $f \in F$ are:

$$w_a(x) = x, \quad w_b(x) = 10, \quad w_c(x) = 10, \quad w_d(x) = x$$

Find a Nash equilibrium for the congestion game associated with the model above.

Problem 2: Braess paradoxon   10 Points

Consider the congestion model $(A, F, (S_i)_{i \in A}, (w_f)_{f \in F})$, similar to one above, for a slightly modified traffic scenario:

where

- the set A of agents is $A = \{1, 2, \dots, 10\}$,

- the set F of facilities (roads) is $F = \{a, b, c, d, e\}$,

- the sets $S_i = S$ of strategies (pathways) is the same for all agents:

$$S = \{ \{a, c\}, \{b, d\}, \{a, e, d\} \}$$

- the cost functions w_f for each facility $f \in F$ are:

$$w_a(x) = x, \quad w_b(x) = 10, \quad w_c(x) = 10, \quad w_d(x) = x, \quad w_e(x) = 0$$

Find a Nash equilibrium for the congestion game associated with the model above.

(Why is it a paradox? *Hint*: Compare both traffic scenarios in Problem 1 and Problem 2.)

Problem 3: ROSENTHAL **potential**   10 Points

Let a finite congestion model $(A, F, (S_i)_{i \in A}, (w_f)_{f \in F})$ be given. For the associated congestion game $\Gamma = (A, S, u)$ with $u_i(s) = \displaystyle\sum_{f \in s_i} w_f(\sigma_f(s))$, the ROSENTHAL *potential* P is defined for each $s \in S$ as

$$P(s) =_{\text{def}} \sum_{f \in \bigcup_{i \in A} s_i} \sum_{k=1}^{\sigma_f(s)} w_f(k)$$

Show that P is a potential function for the game Γ.