University of Konstanz
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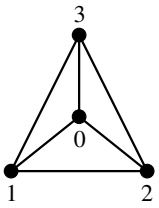
Network Dynamics
Winter 2014/15

# Assignment 2

**Ausgabe:** 29 Oct 2014 **Abgabe:** 05 Nov 2014

## Problem 1: Best-response dynamics **Punkte**

Consider the following graph $G = (V, E)$ together with total orderings of all path sets:




$$
\begin{aligned}
P^0 &: (0) \\
P^1 &: (1,2,0) \succ_1 (1,0) \succ_1 (1,3,0) =_1 (1,3,2,0) =_1 (1,2,3,0) \succ_1 (1) \\
P^2 &: (2,3,0) \succ_2 (2,0) \succ_2 (2,1,0) =_2 (2,1,3,0) =_2 (2,3,1,0) \succ_2 (2) \\
P^3 &: (3,1,0) \succ_3 (3,0) \succ_3 (3,2,0) =_3 (3,1,2,0) =_3 (3,2,1,0) \succ_3 (3)
\end{aligned}
$$

Suppose an initial configuration is

$$\pi(0) = 0, \quad \pi(1) = 2, \quad \pi(2) = 3, \quad \pi(3) = 0.$$

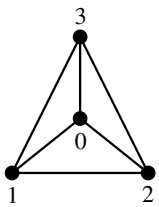
(a) How many update steps are needed to obtain the same configuration again, if the nodes update their best next hops simultaneously?

(b) How many update steps are needed by each node on average to obtain the same configuration again, if the nodes update their best next hops sequentially in the order $(1, 2, 3, 1, 2, 3, 1, 2, 3, \dots)$?

## Problem 2: Best-response dynamics **Punkte**

A configuration $\pi$ is said to be a fixed-point configuration if and only if $\pi = \pi'$ where $\pi'$ is the configuration obtained when all nodes simultaneously update their best next hops given $\pi$.

Consider again the graph $G = (V, E)$:



If possible, design a total ordering of the path sets $P^0, P^1, P^2$, and $P^3$ such that

- there is a fixed-point configuration $\pi_{\text{fix}}$ and
- there is an initial configuration $\pi_{\text{init}}$ such that $\pi_{\text{fix}}$ is never reached assuming that all nodes updates their best next hops simultaneously.

**Problem 3: Valley-free orientations** **10 Points**

Let $G = (V, E)$ be an undirected graph with vertex set $V = \{0, 1, 2, \ldots, n-1\}$. An orientation of an edge $e = \{u, v\}$, $u < v$, in $G$ replaces $e$ by exactly one of the directed edges $u \to v$ or $u \leftarrow v$. An orientation of $G$ consists of orientations of all edges in $G$.

(a) Consider the following path set

$$P = \{\ (1,2,5), (1,0,3,4,7), (7,4,5,8), (4,3,6,7)\ \}$$

Find an orientation of the induced graph $G[P]$ such that all paths are valley-free and the oriented graph is acyclic.

(b) Find a path set $P$ such that an orientation of $G[P]$ cannot satisfy the following both conditions simultaneously:

i. The oriented graph $G[P]$ is acyclic.

ii. All paths in $P$ are valley-free.

Note that, given a path set $P$, the graph $G[P]$ induced by $P$ consists of all vertices that lie on some path in $P$ and of all edges $\{u, v\}$ such that $u$ and $v$ are neighbored on some path in $P$.