University of Konstanz
Department of Computer & Information Science
PD Dr. Sven Kosub / David Schoch

Network Dynamics
Winter 2014/15

Assignment 4

Ausgabe: 12 Nov 2014 **Abgabe:** 19 Nov 2014

Problem 1: Iterated maps **10 Punkte**

Find a map $F : D^n \to D^n$ for each of the sequences $(x_0, x_1, \ldots, x_n, \ldots)$ listed below such that the given sequence is the orbit of x_0 under F, if such an F exists; otherwise prove that no such F exists.

(a) $(0, 1, \sqrt{2}, \sqrt{3}, \ldots, \sqrt{n}, \ldots)$ for $D = \mathbb{R}_{\geq 0}$, $n = 1$

(b) $(1, 1/4, 1/9, 1/16, \ldots, (n+1)^{-2}, \ldots)$ for $D = \mathbb{R}_{\geq 0}$, $n = 1$

(c) $(0, 1, 1, 2, 2, 2, 3, 3, 3, 3, \ldots, \underbrace{n, \ldots, n}_{n+1 \text{ times}}, \ldots)$ for $D = \mathbb{R}_{\geq 0}$, $n = 1$

(d) $(0.a_1a_2a_3\ldots, 0.a_1a_1a_2a_3\ldots, 0.a_1a_1a_1a_2a_3\ldots, \ldots, 0.\underbrace{a_1\ldots a_1}_{n \text{ times}}a_2a_3\ldots, \ldots)$, where $a_i \in \{0, 1\}$ for all $i \in \mathbb{N}$, for $D = [0, 1)$, $n = 1$.

Hint: Consult the example describing the Bernoulli shift in the lecture notes.

(e) $((0, 1), (1, 1), (1, 2), (2, 3), (3, 5), (5, 8), \ldots, (F_n, F_{n+1}), \ldots)$ for $D = \mathbb{N}$, $n = 2$, where F_n is the n-th Fibonacci number, i.e., $F_0 =_{\text{def}} 0$, and $F_n =_{\text{def}} F_{n-1} + F_{n-2}$ for all $n > 0$.

Problem 2: Orbits **10 Punkte**

Let $A \in \mathbb{R}^{2\times 2}$ be a matrix of the following type:

$$A =_{\text{def}} \begin{pmatrix} a & b \\ 0 & a \end{pmatrix}, \qquad a, b \in \mathbb{R}$$

Consider the function $F : \mathbb{R}^2 \to \mathbb{R}^2 : x \mapsto A \cdot x$ (which depends on concrete choices of a, b).

Find all pairs (a, b) of values $a, b \in \mathbb{R}$ such that the orbits of $(-1, 1)$ and $(1, 1)$ under F are disjoint.

Problem 3: Job rotation **10 Points**

In a company, n trainee employees can be assigned to n positions in k departments $D_1, \ldots, D_k$. It is supposed that department D_i has n_i positions, so that $n_1 + \cdots + n_k = n$. Assume that positions are enumerated in ascending order of the departments, i.e., $D_i =_{\text{def}} \{n_{i-1}+1, \ldots, n_i\}$ for all $i \in \{1, \ldots, k\}$, where we set $n_0 =_{\text{def}} 0$. Also assume that trainee employees are enumerated $1, \ldots, n$.

We consider two different networks: Let the set A of items consist of trainees, i.e., $A = \{1, \ldots, n\}$. An assigment is a bijective mapping $\pi : A \to \{1, \ldots, n\}$, i.e., trainee i is currently at position $\pi(i)$. Let the set S of affiliations consist of departments, i.e., $S = \{D_1, \ldots, D_k\}$.

- A two-mode network $x : A \times S \to \{0, 1\}$ is defined as follows:

$$x(i, D_j) = 1 \iff_{\text{def}} \pi(i) \in D_j$$

- A one-mode network $y : A \times A \setminus \{(i, i) | i \in A\} \to \{0, 1\}$ is defined as follows:

$$y(i, j) = 1 \iff_{\text{def}} \text{there is an } \ell \in \{1, \ldots, k\} \text{ such that } \{\pi(i), \pi(j)\} \subseteq D_\ell$$

We assume that the company rotates trainees cyclically to the next position.

(a) Find a total map F that maps, in the two-mode network x, a given assignment to the next assignment.

(b) Find a total map F that maps, in the one-mode network y, a given collaboration configuration to the next one.