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Network Dynamics
Winter 2014/15

Assignment 5

Ausgabe: 19 Nov 2014 **Abgabe:** 26 Nov 2014

Problem 1: Fixed points — 10 Points

Suppose we are given the (symmetrical) interaction domain $\mathcal{I} = \{ \{i,j\} \mid i,j \in A, i \neq j \}$ for a set $A = \{1,\ldots,n\}$. An *open triangle* of the (undirected) network $x : \mathcal{I} \to \{0,1\}$ is formed by three dyads $\{i,j\}$, $\{j,k\}$, and $\{i,k\}$, where i,j,k are pairwise different, such that $x_{\{i,j\}} = x_{\{j,k\}} = 1$ and $x_{\{i,k\}} = 0$.

Define $F : \{0,1\}^{\mathcal{I}} \to \{0,1\}^{\mathcal{I}}$ to be the map that closes all open triangles of a given network x, i.e., if dyads $\{i,j\}$, $\{j,k\}$, and $\{i,k\}$ form an open triangle in the network x then in network $F(x)$, it holds that $F(x)_{\{i,j\}} = F(x)_{\{j,k\}} = F(x)_{\{i,k\}} = 1$.

How many fixed points of F (depending on n) are associated with connected graphs? Prove your hypothesis.

Problem 2: Basins of attraction — 10 Points

Let $F : D \to D$ be an arbitrary total mapping over a finite domain D. Consider the following two definitions for some non-empty set $E \subseteq D$:

- E is said to be a *weak invariant set* of F if and only if $F(E) \subseteq E$.
- E is said to be a *strong invariant set* of F if and only if $F^k(E) \subseteq E$ for all $k \in \mathbb{N}$.

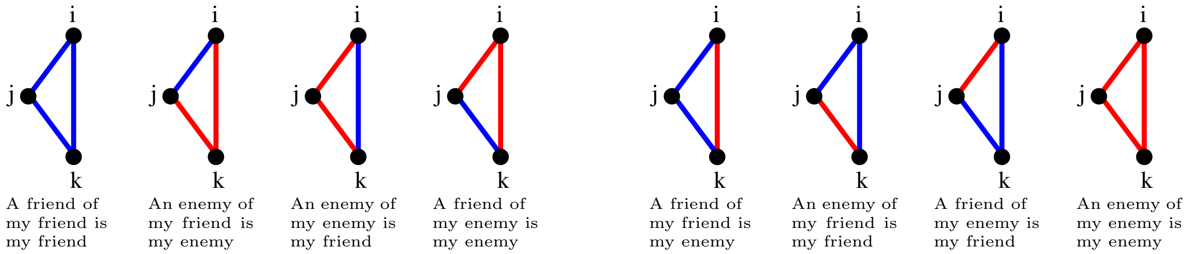
Which type of an invariant set–weak or strong–provides a guarantee for containment of a limit cycle? Prove your hypothesis.

Hint: Note that $G(E) =_{\text{def}} \{ G(x) \mid x \in E \}$ for a mapping $G : D \to D$ and a set $E \subseteq D$.

Problem 3: Conflict networks — 10 Points

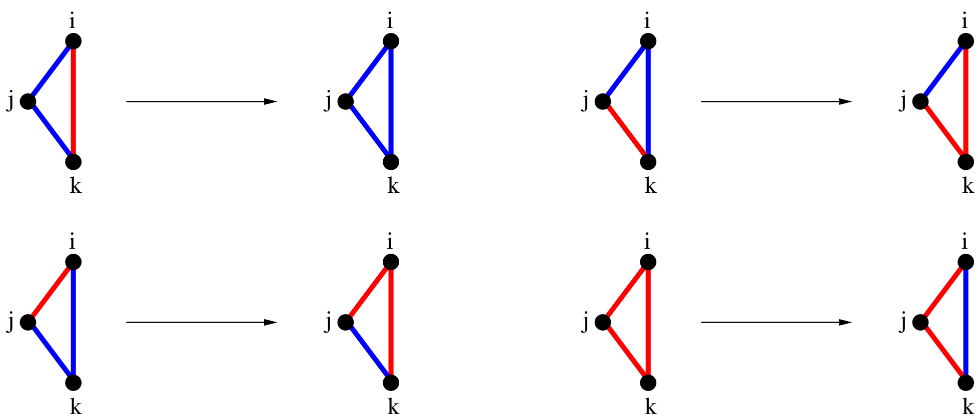
Let $\mathcal{I} \subseteq A \times A$ be an arbitrary symmetrical interaction domain for a finite set $A = \{1,\ldots,n\}$ of actors. We consider a (categorical) network attribute $x : \mathcal{I} \to \{\mathsf{friend}, \mathsf{enemy}\}$. Suppose we are given a triple (i,j,k) of actors $i,j,k \in A$ such that $\{i,j\},\{j,k\},\{i,k\} \in \mathcal{I}$. Notice that the order of the actors in the triple is important in the forthcoming.

The network attribute restricted to the triple (i,j,k) has one of the following representations as a graph, where a blue edge represents attribute value friend and a red edge represents attribute value enemy.



Note that, when interpreting the graphs, according to the triple (i, j, k), actor i is the focal actor and actor j is closer to actor i than actor k.

The left group of four graphs represents stable situations whereas the right group of four graphs represents instable situations. In these situations, instability is resolved according to the following replacement rules:



Now, suppose a concrete interaction domain over a set $A = \{1, 2, 3, 4\}$ of actors is given:

$$\mathcal{I} =_{\text{def}} \{ \ \{1,2\}, \{2,3\}, \{3,4\}, \{4,1\}, \{2,4\} \ \}$$

$\mathcal{I}$ consists of the two triangles $\{1, 2, 4\}$ and $\{2, 3, 4\}$.

We consider a total mapping $F : \{\mathsf{friend}, \mathsf{enemy}\}^{\mathcal{I}} \to \{\mathsf{friend}, \mathsf{enemy}\}^{\mathcal{I}}$ which assigns to a network $x : \mathcal{I} \to \{\mathsf{friend}, \mathsf{enemy}\}$ a network $F(x)$ where stable triangles of x are maintained and instable triangles of x are replaced according to the replacement rules. Here, the triangles are understood as triples $(2, 1, 4)$ and $(2, 3, 4)$. In the case that replacement rules for an edge are inconsistent, value enemy beats value friend.

Draw the state graph $\Gamma(F)$ of F. (Note that $\Gamma(F)$ consists of 32 nodes.)