UNIVERSITY OF KONSTANZ
DEPARTMENT OF COMPUTER & INFORMATION SCIENCE
PD Dr. Sven Kosub / David Schoch

Network Dynamics
Winter 2014/15

Assignment 6

Ausgabe: 26 Nov 2014 **Abgabe:** 03 Dec 2014

Problem 1: Phase space — 10 Points

Consider the following boolean map

$$F : \{0,1\}^3 \to \{0,1\}^3 : (x_1, x_2, x_3) \mapsto (x_1 \oplus (x_2 \vee x_3), 1 \oplus x_1 \oplus x_2, x_2 \oplus x_3),$$

where $\oplus$ denotes the XOR function.

(a) Find a state with maximum visiting probability.

(b) Find the probability that a random orbit of F has a fixed point.

Problem 2: Subdynamics — 10 Points

Let $F : D^n \to D^n$ be a map. A subset $L \subseteq \{1, \ldots, n\}$ of size $\|L\| = m$ is called *level of F* if and only if $L = \emptyset$ or there is a map $G : D^m \to D^m$ such that for all $x \in D^n$,

$$\pi(F(x)) = G(\pi(x)),$$

where $\pi : D^n \to D^m$ is the projection that maps $(x_1, \ldots, x_n)$ to those m components indexed by the set L, i.e., $\pi(x_1, \ldots, x_n) = (x_{i_1}, \ldots, x_{i_m})$ and $L = \{i_1, \ldots, i_m\}$.

Prove or disprove the following statements:

(a) If L_1 and L_2 are levels of F then $L_1 \cap L_2$ is a level of F.

(b) If L_1 and L_2 are levels of F then $L_1 \cup L_2$ is a level of F.

Problem 3: Local maps — 10 Points

Draw the directed interdependence graph of the iterated map $F : \{0,1\}^4 \to \{0,1\}^4$ given by the following truth table:

(x_1, x_2, x_3, x_4)	$F(x_1, x_2, x_3, x_4)$
0000	1001
0001	1001
0010	1000
0011	1001
0100	1001
0101	1001
0110	1000
0111	1001
1000	1001
1001	1001
1010	1000
1011	1001
1100	1011
1101	1011
1110	1010
1111	1011