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Network Dynamics
Winter 2014/15

Assignment 8

Ausgabe: 10 Dec 2014 **Abgabe:** 17 Dec 2014

Problem 1: Functional equivalence — 10 Points

A cycle C_4 of length 4 has 14 different acyclic orientations.

Find a set $L = \{f_1, f_2, f_3, f_4\}$ of local transition functions compatible with C_4 such that there are exactly 14 different global network maps (or state graphs, respectively).

Problem 2: Update graph — 10 Points

A graph $G = (V, E)$ is said to be *sequentially equicausal* if and only if for all permutations $\pi, \pi' : V \to V$, it holds that $\|[\pi]_U\| = \|[\pi']_U\|$ with respect to the update graph $U = U(V, E)$.

Find the class of all sequentially equicausal graphs.

Problem 3: Acyclic orientations — 10 Points

For $n, m \in \mathbb{N} \setminus \{0\}$, an (n, m)-*mesh* $M_{n,m} = (V_{n,m}, E_{n,m})$ is an undirected graph defined as:

$$\begin{aligned} V_{n,m} &=_{\text{def}} \{1, \ldots, n\} \times \{1, \ldots, m\} \\ E_{n,m} &=_{\text{def}} \{\ \big((x, y), (x', y')\big) \mid x, x' \in \{1, \ldots, n\}, y, y' \in \{1, \ldots, m\}, \text{ and } \\ &\qquad\qquad |x - x'| + |y - y'| = 1\ \} \end{aligned}$$

In other words, $M_{n,m}$ consists of vertices arranged in n rows and m columns, and edges connect consecutive vertices along each row and consecutive vertices along each column. For instance, the cycle C_4 can be identified with a $(2, 2)$-mesh $M_{2,2}$.

How many acyclic orientations are there for an $M_{2,3}$?