UNIVERSITY OF KONSTANZ
DEPARTMENT OF COMPUTER & INFORMATION SCIENCE
PD Dr. Sven Kosub / David Schoch

Network Dynamics
Winter 2014/15

Assignment 9

Ausgabe: 17 Dec 2014 **Abgabe:** 14 Jan 2015

Problem 1: Chromatic polynomial — 10 Points

Find the chromatic polynomial P_n for a cycle of length $n \in \mathbb{N} \setminus \{0, 1\}$.

Problem 2: Chromatic polynomial — 10 Points

Prove or disprove all trees on n vertices have the same chromatic polynomial; in case of an affirmative answer try to find the polynomial.

Problem 3: Games with utilities — 10 Points

Consider a group $A = \{1, \ldots, n\}$ of n persons being pairwise friends. A person $i \in A$ wants to spend time with each friend $j \in A$ solely but has only limited amount t_i of spare time. The problem is how to distribute the time among the friends.

We formulate the problem as the following game $\Gamma = (A, S, u)$ with utilities:

- $A = \{1, \ldots, n\}$
- $S = S_1 \times \cdots \times S_n$ where

$$S_i = \{\ (s_{i1}, \ldots, s_{in}) \mid s_{ij} \geq 0 \text{ and } s_{i1} + s_{i2} + \cdots + s_{in} = t_i\ \}$$

  for each $i \in A$
- $u = (u_1, \ldots, u_n)$ where $u_i(s_1, \ldots, s_n) = \displaystyle\sum_{\substack{j=1 \\ i \neq j}}^{n} \min\{s_{ij}, s_{ji}\}$

The interpretation of the utility function is that the mutual time of two persons depends on the time reciprocally made available by both persons and that time spended without a friend is worthless.

Find a Nash equilibrium of the game Γ for arbitrary times $t_1, \ldots, t_n > 0$.

Merry Christmas and a Happy New Year!