University of Konstanz
Department of Computer & Information Science
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Network Dynamics
Winter 2014/15

Assignment 11

Ausgabe: 21 Jan 2015 **Abgabe:** 28 Jan 2015

Problem 1: Potential games — 10 Points

In the following, apply the characterization of potential games by closed paths, as given in the lecture.

(a) Prove or disprove that the following bimatrix game

$$\Gamma = \begin{pmatrix} (1,3) & (2,3) & (3,3) \\ (2,2) & (3,1) & (1,3) \\ (3,3) & (1,1) & (2,2) \end{pmatrix}$$

is a potential game.

(b) Suppose you are given an $n \times k$-game $\Gamma = (A, S, u)$, i.e., a game with n agents each of which has k strategies at hand. Depending on n and k, how many closed paths do have to check in order to decide whether Γ is a potential game.

Problem 2: Ordinal potential games — 10 Points

Prove or disprove that the following bimatrix game

$$\Gamma = \begin{pmatrix} (1,2) & (2,3) & (3,1) \\ (2,2) & (3,1) & (1,3) \\ (3,3) & (1,1) & (2,2) \end{pmatrix}$$

is an ordinal potential game.

Hint: Check the Finite Improvement Property for Γ.

Problem 3: Improvement paths — 10 Points

A game $\Gamma = (A, S, u)$ is called *degenerate* iff there exist $i \in A$, $s_i, s_i' \in S_i$, $s_i \neq s_i'$, and $s_{-i} \in S_{-i}$ such that $u_i(s_i, s_{-i}) = u_i(s_i', s_{-i})$; otherwise, Γ is called *nondegenerate*.

Find a nondegenerate 2×3-game Γ (i.e., a game with 2 players each of which has 3 strategies) such that the length of the longest (finite) improvement path of Γ is minimum among all nondegenerate 2×3-games.

Hint: Consult the proof of Theorem 6.10 in the lecture notes (v4.7 or higher), and construct an appropriate strict order for Γ.