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Department of Computer & Information Science
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Network Dynamics
Winter 2014/15

Assignment 7

Ausgabe: 03 Dec 2014 **Abgabe:** 10 Dec 2014

Problem 1: Global transition function — 10 Points

Consider the following set $L = \{f_1, f_2, f_3\}$ of boolean local transition functions

$$\begin{aligned} f_1(x_1, x_2, x_3) &=_{\text{def}} x_1 \oplus (x_2 \vee x_3) \\ f_2(x_1, x_2, x_3) &=_{\text{def}} 1 \oplus x_1 \oplus x_2 \\ f_3(x_1, x_2, x_3) &=_{\text{def}} x_2 \oplus x_3 \end{aligned}$$

where $\oplus$ denotes the XOR function.

Determine the global transition function for L.

Problem 2: Global network maps — 10 Points

Suppose $A = \{1, \ldots, n\}$ is a set of n actors. An update schedule $\alpha : \{1, \ldots, T\} \to \mathcal{P}(A)$ is called *sequential* if and only if $T = n$ and there is a permutation $\pi : \{1, \ldots, n\} \to \{1, \ldots, n\}$ such that $\alpha(i) = \{\pi(i)\}$. A sequential update schedule is usually identified with a permutation.

Now consider again the same set L of boolean local transition function as in Problem 1.

How many of the six different permuations π of the set $A = \{1, 2, 3\}$ induce different global network maps $\mathbf{F}_{(L,\pi)}$?

Problem 3: Fixed points — 10 Points

Suppose you are given a propositional formula $H = H(x_1, \ldots, x_n)$ in conjunctive normal form, i.e.,

$$H = \bigwedge_{i=1}^{m} (L_{i,1} \vee L_{i,2} \vee L_{i,3}),$$

where $L_{i,j} \in \{x_1, \ldots, x_n, \neg x_1, \ldots, \neg x_n\}$ are literals. You may assume that no variable occurs twice in a clause $(L_{i,1} \vee L_{i,2} \vee L_{i,3})$.

Find a possibly sparse undirected graph $G = (V, E)$ and a set L of boolean local transition functions compatible with G, both depending on H, such that the following is true:

$$\text{there is a satisfying truth-assigment for } H \iff \mathbf{F}_{(L,\alpha)} \text{ has a fixed point}$$

where $\alpha : 1 \mapsto V$ is a synchronous update schedule.