University of Konstanz
Algorithmics
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Network Analysis
Summer 2014

Assignment 4

Issue date: 15 May 2014 **Due date:** 22 May 2014, 11:00
It is explicitly recommended to solve exercises in groups of two.

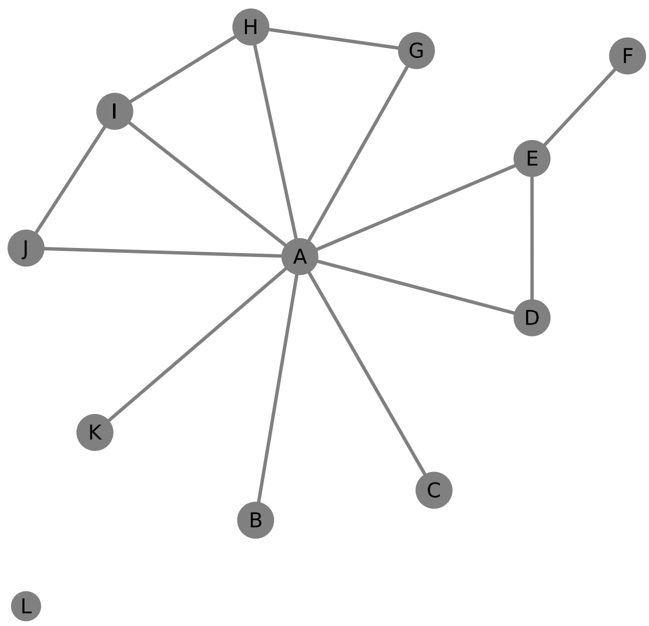
Exercise 1: Neighborhood Inclusion 2+1 Points

(a) Let $G = (V, E)$ be an arbitrary graph. Asume, there are vertices v and w sucht that: $N(v) \setminus \{w\} \supseteq N(w) \setminus \{v\}$.
Show that the following inequality holds:

$$\sum_{u \in V} dist(v, u) \leq \sum_{u \in V} dist(w, u),$$

where $dist(v, u)$ is the length of the shortest path between v and u.

(b) List all pairs of vertices in Wolfes monkey network that are comparable by neighborhood inclusion



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Exercise 2: Threshold Graphs **3+3+3+2 Points**

(a) *An* ***alternating 4-cycle*** *of a Graph G=(V,E) is a configuration consisting of distinct vertices a, b, c, d such that* $\{a, b\}, \{c, d\} \in E$ *and* $\{a, c\}, \{b, d\} \notin E$*. Depending on the presence or absence of the edges* $\{a, d\}$ *and* $\{b, c\}$*, the vertices of an alternating 4-cycle induce a* P_4*,* C_4 *or* $2K_2$*.*
Prove the following statement:
G is a threshold graph $\iff$ G does not contain an alternating 4-cycle.

(b) *A* ***threshold sequence*** *is a degree seqence of a threshold graph.*
Let $k = \max\{i : d_i \geq i - 1\}$. Prove, that every threshold sequence satisfies the first k Erdős-Gallai inequalities with equality.

(c) Let b be an arbitrary binary sequence of length n with $b_1 = 0$ and $b_n = 1$, such that it can be interpreted as a construction sequence of a connected threshold graph G. Give an analytic formula, to calculate the degree sequence of G.
HINT: If $b_i = b_{i+1}$ then $deg(i) = deg(i + 1)$

(d) Decide if the following graph is a threshold graph. If so, give the ordering of the vertices, the binary sequence that produces the graph and assign vertex weights as well as the threshold.