University of Konstanz
Algorithmics
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Network Analysis
Summer 2014

Assignment 5

Issue date: 22 May 2014 **Due date:** 29 May 2014, 11:00
It is explicitly recommended to solve exercises in groups of two.

Exercise 1: Triangles and Clustering Coefficient 3+3+2 Points

(a) You are given the assumptions of Exercise 2c) of the last assignment sheet. Derive an analytic formula to calculate the number of triangles each node in a threshold graph participates in.

(b) Calculate the clustering coefficient for each node in an arbitrary threshold graph. Under which conditions is the average clustering coefficient close to one or zero? In general: Would you consider a threshold graph to be a "small world"?

(c) The table shows the triad census for three different networks. Assign each network to one of the following classes: $\mathcal{G}(n, p)$, $(n, 1, r)$-torus and preferential attachment. Give an explanation for your choices. Further determine the n and p (one decimal place) for the $\mathcal{G}(n, p)$ network and the n and r for the $(n, 1, r)$-torus.

	a	b	c
T[0]	116200	119685	123889
T[1]	43000	37936	28744
T[2]	1500	3917	8415
T[3]	1000	162	652

[please turn over]

Exercise 2: Strong Connectivity and DFS **2+4 Points**

(a) Determine the strongly connected components of the directed graph $G = (V, E)$ with $V = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $E = \{(1, 5), (1, 8), (2, 1), (2, 6), (4, 2), (4, 5), (4, 7), (5, 8), (6, 2), (7, 3), (7, 4), (8, 1)\}$ and draw the corresponding DAG.

(b) Let $G = (V, E)$ be a directed graph and $T(G)$ a subgraph of G induced by the tree edges of a depth-first search. Hence, $T(G)$ is a forest consisting of one or more rooted trees. Let $D(v)$ denote for each vertex $v \in V$ the number of vertices in the subtree of $T(G)$ rooted at v (v included).
Show that the subtree rooted at $v \in V$ contains $w \in V$ if and only if

$$DFS(v) \leq DFS(w) < DFS(v) + D(v).$$

Please submit your answers electronically to teaching assistant David (david.schoch@uni-konstanz.de).