University of Konstanz
Algorithmics
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Network Analysis
Summer 2014

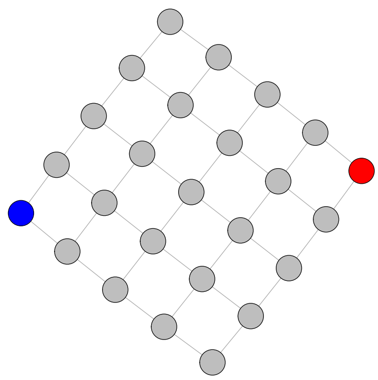
Assignment 7

Issue date: 05 June 2014 **Due date:** 12 June 2014, 11:00
It is explicitly recommended to solve exercises in groups of two.

Exercise 1: Spectra and Walks — 2+3 Points

(a) The multiset of eigenvalues of a graph is called the *spectrum* of a graph. Calculate the spectrum of a complete graph K_n.

(b) The figure below shows a simplified version of the street map of Manhattan (NYC), where edges are the streets and vertices the crossings. Imagine a drunkard exits a bar (blue vertex) and tries to walk home (red vertex). However, due to his dazed state, he can not remember the way home and so starts to wander around through the streets, picking a random direction at each crossing. After what felt like an eternity, he finally reaches home and sleeps off his drunkenness. In the morning, he tries to remember how long it took him to reach home. However, he can't figure it out, but at least he thinks, that he past an even number of crossings. Give a proof or explain why this is indeed true.



[please turn over]

Exercise 2: Network Position and Centrality 5+4 Points

(a) Let $x : \mathcal{D} \rightarrow \mathcal{R}$ be a binary symmetric network with simple undirected graph $G = (V, E)$. The detailed positions are described by $\textcircled{i} = (x_{ik})_{k \in V} \in \{0, 1\}^n$ where

$$x_{ik} = \begin{cases} 1 & \text{if } \{i, k\} \in E \\ 0 & \text{otherwise .} \end{cases}$$

Show that

$$\textcircled{i} \succeq \textcircled{j} \implies betweenness(i) \geq betweenness(j)$$

(b) You are given the the assumptions of a). Further, let $\#P_2(i)$ be the number of 2-paths starting in i (i.e. $i \rightarrow k \rightarrow l$, $k \neq l \neq i \in V$).
Prove or give a counterexample:

$$\text{If } \textcircled{i} \succeq \textcircled{j}, \text{ then } \#P_2(i) \geq \#P_2(j)$$

Please submit your answers electronically to teaching assistant David (david.schoch@uni-konstanz.de).