University of Konstanz
Algorithmics
Prof. Dr. U. Brandes, PD Dr. S. Kosub, Habiba, D. Schoch

Network Analysis
Summer 2014

# Assignment 11

**Issue date:** 03 July 2014 **Due date:** 10 July 2014, 11:00
It is explicitly recommended to solve exercises in groups of two.

## Exercise 1: Cores **5+5 Points**

(a) A simple undirected graph $G = (V, E)$ is *$k$-colorable*, if there exists a mapping $f : V \to \{1, \ldots, k\}$, such that $f(v) \neq f(w)$ for all $\{v, w\} \in E$, i.e. adjacent nodes are colored differently.

The smallest $k \in \mathbb{N}$ for which $G$ is $k$-colorable is called its *chromatic number* $\chi(G)$. Prove that $\chi(G) \leq \mathit{core}(G) + 1$.

(b) Let $G = (V, E, f)$ be a simple, undirected, weighted graph with $f : E \to \mathbb{R}_0^+$. A subgraph $C_t(G) \subseteq G$ is called *weighted $t$-core*, if it is inclusion-maximal and it holds that

$$\sum_{w \in N_{C_t(G)}(v)} f(\{v, w\}) \geq t \qquad \text{for all } v \in C_t(G).$$

Show that the weighted $t$-core of a graph is unique and provide a *simple* algorithm for calculation.

[please turn over]

**Exercise 2: Plexes** **5+5 Points**

In lecture you studied the concept of core structure in graphs. A $k$-core is defined as:

> Let $G = (V, E)$ be any undirected graph and let $k \in \{1, \ldots, n-1\}$ be a natural number. A subset $U \subseteq V$ is said to be an $k$-core if and only if $\delta(G[U]) \geq k$.

Now, consider a complementary concept of the core structure called *Plexes*. A $k$-plex of a graph is defined as:

> Let $G = (V, E)$ be any undirected graph and let $k \in \{1, \ldots, n-1\}$ be a natural number. A subset $U \subseteq V$ is said to be a $k$-plex if and only if $\delta(G[U]) \geq |U|$ - $k$.

Prove:

(a) If $V$ is a $k$-plex with $k < \frac{n+2}{2}$, then $diameter(G) \leq 2$ and, if additionally $n \geq 4$, $G$ is 2-*edge–connected*.

(b) If $V$ is a $k$-plex with $k \geq \frac{n+2}{2}$, then $diameter(G) \leq 2k - n + 2$.

Recall:

- *diameter*$(G)$ is the length of the *longest shortest path* of a graph $G$.
- A graph $G$ is $k$-edge–connected if it remains connected whenever fewer than $k$ edges are removed.

**Please submit your answers electronically to teaching assistant Habiba (habiba@uni-konstanz.de).**