University of Konstanz
Department of Computer & Information Science
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Assignment 1

Post Date: 24 Oct 2012 **Due Date:** 31 Oct 2012, 14:30
You are permitted and encouraged to work in groups of two.

Problem 1: Growth of Functions — 6 Points

Rank the functions

$$n \log n,\ \binom{2n}{4},\ 2^n,\ 4n^4,\ n^n,\ 16^{\log_2 n},\ \log(n!)$$

by increasing order of growth, i.e., find an order $f_1, \ldots, f_7$ with $f_1 \in \mathcal{O}(f_2), \ldots, f_6 \in \mathcal{O}(f_7)$, and proof the correctness of your ranking.

Problem 2: Recurrence Equations — 4 Points

Give a Θ-bound for the following recurrences:

(a) $T_1(n) = 2 \cdot T_1\left(\frac{n}{2}\right) + \sqrt{n}, \quad T_1(1) = 1$

(b) $T_2(n) = T_2(n-1) + 2(n-1), \quad T_2(1) = 1$

Problem 3: Threshold — 4 Points

Give the greatest $a \in \mathbb{N}$ such that Algorithm A with running time

$$T_A(n) = a \cdot T_A\left(\frac{n}{4}\right) + n^2$$

is asymptotically faster than Algorithm B with running time

$$T_B(n) = 7 \cdot T_B\left(\frac{n}{2}\right) + n^2.$$

[please turn over]

Problem 4: Divide and Conquer **6 Points**

Let be given n points in the plane. You may assume for simplicity that no two points have the same x-coordinate. Develop a divide-and-conquer algorithm that finds a pair of points with the smallest Euclidean distance between them. Analyze the running time of your algorithm that should be in $\mathcal{O}(n \log n)$.

Hint: It may be helpful to think first about the 1D version of the problem. How can the merge step be realized in 2D?