University of Konstanz
Department of Computer & Information Science
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Design and Analysis of Algorithms
WS 2012/2013

Assignment 5

Post Date: 21 Nov 2012 **Due Date:** 28 Nov 2012, 14:30
You are permitted and encouraged to work in groups of two.

Problem 1: Minimum Spanning Tree — 6 Points

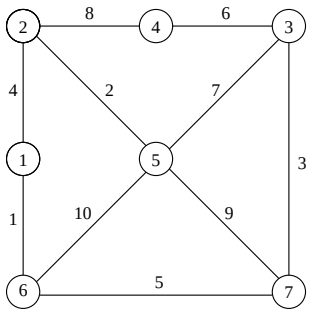
Find an MST for the graph on the right using

(a) the coloring method of Tarjan,

(b) the algorithm of Kruskal, and

(c) the algorithm of Prim.

Indicate in each step the edge that has to be colored together with the corresponding color.



Problem 2: Unique Minimum Spanning Trees — 6 Points

(a) Prove: Let G be a connected graph with real-valued edge weights. If for each cut the crossing edge with the lightest weight is unique, then G has a unique minimum spanning tree.

(b) Does the inverse of the implication in (a) hold?

(c) Prove or disprove: If in a connected graph G with real-valued edge weights all edges have pairwise distinct edge weights, then G has a unique minimum spanning tree.

(d) Does the inverse of the implication in (c) hold?

Problem 3: Trees — 8 Points

(a) Show the equivalence of the following properties for an undirected graph $G = (V, E)$:

i. G is connected and acyclic.

ii. There is a unique simple path between any two vertices in G.

iii. G is connected and for each $e \in E$ the subgraph $G' = (V, E \setminus \{e\})$ is disconnected.

iv. G is acyclic and for each $e \in \binom{V}{2} \setminus E$ the graph $G' = (V, E \cup \{e\})$ contains a cycle.

(b) Proof by induction that a tree T with $|V|$ vertices contains $|V| - 1$ edges.