UNIVERSITY OF KONSTANZ
DEPARTMENT OF COMPUTER & INFORMATION SCIENCE
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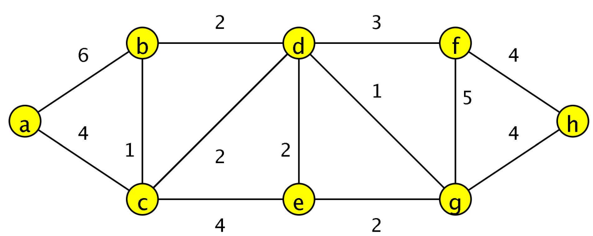
Design and Analysis of Algorithms
WS 2012/2013

Assignment 6

Post Date: 28 Nov 2012 **Due Date:** 05 Dec 2012, 14:30
You are permitted and encouraged to work in groups of two.

Problem 1: Minimum Cut — 8 Points

(a) Find a minimum cut for the following graph with the algorithm of Stoer and Wagner. Start with vertex a and comment each step of the algorithm.



(b) Which conclusion in the proof of correctness of the algorithm of Stoer and Wagner requires nonnegative edge weights? Find an example graph with negative edge weights such that the algorithm of Stoer and Wagner does not find a minimum cut.

Problem 2: Crossing Cuts — 8 Points

Two cuts $(S, V \setminus S)$ and $(T, V \setminus T)$ in a graph $G = (V, E)$ *cross* if no set of sets $A := S \cap T$, $B := S \setminus T$, $C := T \setminus S$, and $D := V \setminus (S \cup T)$ is empty. Let $c : E \to \mathbb{R}_0^+$ be a weight function.

(a) Let $(S, V \setminus S)$ and $(T, V \setminus T)$ be crossing minimum cuts in G with weight λ. Show that

i. $c(A, D) = c(B, C) = 0$ and

ii. $c(A, B) = c(B, D) = c(D, C) = c(C, A) = \frac{\lambda}{2}$.

(b) Let v and w be two adjacent vertices with $c(\{v, w\}) > 0$. Prove that the set of all minimum cuts of G that separate v and w does not contain any crossing cuts.

[please turn over]

Problem 3: Vertex Capacities **4 Points**

Let $((V, E); s, t; c)$ be an extended flow network where not only edge capacities, but also vertex capacities are constrained, i. e., $c : E \cup V \to \mathbb{R}_0^+$ and a flow $f : E \to \mathbb{R}_0^+$ must satisfy, in addition to the edge capacity constraint and the flow conservation constraint, the following vertex capacity constraint

$$\sum_{w:(w,v)\in E} f(w, v) \leq c(v) \text{ for all } v \in V.$$

Show that determining a maximum flow in a network with additional vertex capacities can be reduced to determining a maximum flow in a network with only edge capacities.