

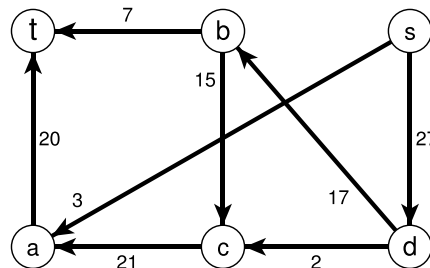
## Assignment 7

**Post Date:** 05 Dec 2012    **Due Date:** 12 Dec 2012, 14:30  
 You are permitted and encouraged to work in groups of two.

### Problem 1:

**8 Points**

Consider the following flow network where the numbers at the edges denote their capacity:



- Find a maximum flow and a minimum  $s$ - $t$  cut using the algorithm of Ford & Fulkerson. Indicate in each step the augmenting path and the value of the augmentation.
- Find a maximum flow using the algorithm of Goldberg and Tarjan. Indicate in each step:
  - the selected active vertex,
  - the selected basic operation / the involved vertices,
  - depending on the operation, the value  $\Delta$  for the corresponding edge and the new flow excess, or the  $h$ -labels for the vertices where the labels were changed.

### Problem 2: Admissible Paths

**4 Points**

Let  $h : V \rightarrow \mathbb{N}_0$  be a height function that is compatible with a preflow  $f$  in a flow network  $(V, s, t, c)$ . An *admissible arc* is an arc  $(v, w)$  of the residual network with  $h(v) = h(w) + 1$ . An *admissible path* is a path that consists of admissible arcs only. Show that for any vertex  $v \in V$  an admissible  $v$ - $t$ -path is a shortest  $v$ - $t$ -path in the residual network.

**Problem 3: Santa Claus Problem****8 Points**

In order to have enough presents for all children, Santa Claus prepares presents all year round. At the beginning of December he obtains wish lists from all the children. Now Santa Claus tries to assign each child a present such that as many children as possible obtain a present they are happy with.

- (a) Formulate the problem as a maximum flow problem. Show that an optimum solution of Santa Claus' problem can indeed be constructed from a maximum flow in your flow network. How fast can an optimum assignment of presents be found?
- (b) For a child  $c$  let  $W(c)$  be the set of presents on its wish list. For a set  $C$  of children let  $W(C) = \bigcup_{c \in C} W(c)$ . Use the max-flow min-cut theorem to show that Santa Claus can assign the presents such that all children receive a present they like if and only if there is no subset  $C$  of children with  $|W(C)| < |C|$ .