University of Konstanz
Department of Computer & Information Science
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Design and Analysis of Algorithms
WS 2012/2013

Assignment 9

Post Date: 19 Dec 2012 **Due Date:** 9 Jan 2013, 14:30
You are permitted and encouraged to work in groups of two.

Throughout these assignments, we denote by *boundary function* the function

$$B(j) = \begin{cases} -1, & j = 0 \\ |\partial(P[1,\ldots,j])| = \mathrm{suf}_P(P[2,\ldots,j]), & j = 1,\ldots,m \end{cases}$$

assigning to the length of a prefix of a pattern $P[1,\ldots,m]$ the length of its proper boundary.

Problem 1: Knuth-Morris-Pratt and Boyer-Moore — 8 Points

(a) Compute the boundary function for the pattern

$$P = \text{ababbabbababbababbabb}.$$

(b) Compute the bad character function and the good suffix function for the alphabet $\Sigma = \{0,1,2,3\}$ and the pattern

$$P = 0101101201.$$

Problem 2: Transition Function — 8 Points

Let δ be the transition function of a pattern $P[1,\ldots,m]$.

(a) Show that $\delta(q,a) = \delta(B[q],a)$ for any $a \in \Sigma$ and $0 < q \leq m$ with $q = m$ or $P[q+1] \neq a$.

(b) Give an $\mathcal{O}(m|\Sigma|)$-time algorithm for computing the transition function δ corresponding to a given pattern P of length m.

Problem 3: Repetition Factor — 4 Points

Let P be a pattern of length m. For a $q = 1,\ldots,m$ let

$$\rho(q) = \max\{r;\ P[1,\ldots,q] = x^r \text{ for some } x \in \Sigma^*\}.$$

Prove or disprove that $\rho(q) > 1$ if and only if there is an $i > 0$ with $B^i(q) > 0$ and

$$q - B^i(q) = \frac{B^i(q)}{\rho(B^i(q))}.$$

Merry Christmas and a happy New Year!