University of Konstanz
Department of Computer & Information Science
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Design and Analysis of Algorithms
WS 2012/2013

Assignment 12

Post Date: 23 Jan 2013 **Due Date:** 30 Jan 2013, 14:30
You are permitted and encouraged to work in groups of two.

Problem 1: Carpenter's Problem — 4 Points

In the attic you find some wood, more precisely $24m^2$ of plywood and $20m^2$ of pine wood. You decide to construct some tables and some book cases with it. For a table you need $4m^2$ of plywood and $2m^2$ of pine wood. For a book case you need $3m^2$ of plywood and $4m^2$ of pine wood. You can sell a table to the local furniture store for 700 Euro and a book case for 600 Euro.

Formulate a linear program for the problem of deciding how you can most profitably use the wood.

Problem 2: Standard and Slack Form — 8 Points

Consider the following linear program L_1.

$$\text{minimize} \qquad -x_1 + x_2$$

subject to

$$\begin{array}{rcrcr} -x_1 & + & 2x_2 & \geq & -2 \\ x_1 & + & x_2 & \leq & 5 \\ & & x_1, x_2 & \geq & 0 \end{array}$$

(a) Convert L_1 into standard form.

(b) Visualize the LP by drawing the feasible region and two lines for different objective values.

(c) Convert the LP into slack form.

Among (d) and (d') you may choose to solve one.

(d) Solve the LP using the Simplex algorithm. Give the slack form after each pivoting-step. Visualize each intermediate step in your solution to (b) by drawing the point (x_1, x_2) that is obtained by setting the non-basic variables to zero.

[please turn over]

(d') Consider the linear program L_2

$$\text{maximize} \qquad x_1 - x_2 - x_4$$

subject to

$$\begin{array}{rcrcrcrcl} x_1 & - & 2x_2 & - & x_3 & - & 2x_4 & \leq & 0 \\ x_1 & + & x_2 & + & 2x_3 & + & x_4 & \leq & 4 \\ x_1 & + & x_2 & + & x_3 & & & \leq & 7 \\ & & & & & & x_1, x_2, x_3, x_4 & \geq & 0 \end{array}$$

Show that $L_2 \leq L_1$.

Problem 3: Santa Claus Revisited **4 Points**

Formulate the Santa Claus Problem, i.e., Problem 3 of Assignment 7, as a linear program.

Problem 4: Bounded Linear Programs **4 Points**

Prove or disprove the following statements.

(a) The feasible region of a linear program is bounded if its optimal objective value is finite.

(b) For a feasible linear program there is at most one solution with optimal objective value.