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Network Dynamics
Winter 2013/14

Assignment 10

Ausgabe: 8 Jan 2013 **Abgabe:** 15 Jan 2014

Problem 1: Local maps — 10 Points

Draw the directed interdependence graph of the iterated map $F : \{0,1\}^4 \to \{0,1\}^4$ given by the following truth table:

(x_1, x_2, x_3, x_4)	$F(x_1, x_2, x_3, x_4)$
0000	1001
0001	1001
0010	1000
0011	1001
0100	1001
0101	1001
0110	1000
0111	1001
1000	1001
1001	1001
1010	1000
1011	1001
1100	1011
1101	1011
1110	1010
1111	1011

Problem 2: Games with utilities — 10 Points

Consider a group $A = \{1, \ldots, n\}$ of n persons being pairwise friends. A person $i \in A$ wants to spend time with each friend $j \in A$ solely but has only limited amount t_i of spare time. The problem is how to distribute the time among the friends.

We formulate the problem as the following game $\Gamma = (A, S, u)$ with utilities:

- $A = \{1, \ldots, n\}$
- $S = S_1 \times \cdots \times S_n$ where

  $$S_i = \{\ (s_{i1}, \ldots, s_{in}) \mid s_{ij} \geq 0 \text{ and } s_{i1} + s_{i2} + \cdots + s_{in} = t_i\ \}$$

  for each $i \in A$

- $u = (u_1, \ldots, u_n)$ where $u_i(s_1, \ldots, s_n) = \sum_{\substack{j=1 \\ i \neq j}}^{n} \min\{s_{ij}, s_{ji}\}$

The interpretation of the utility function is that the mutual time of two persons depends on the time reciprocally made available by both persons and that time spended without a friend is worthless.

Find a Nash equilibrium of the game Γ for arbitrary times $t_1, \ldots, t_n > 0$.

Problem 3: Netlogo **10 Points**

Consider again the game $\Gamma = (A, S, u)$ specified in Problem 2 above. For each person $i \in A$ define the local transition function $f_i : S \to S_i$ as follows: Set $\Delta_{ij} =_{\text{def}} s_{ji} - s_{ij}$ and $p_i =_{\text{def}} \arg\max_{j \neq i} \Delta_{ij}$. Define $f_i(s_1, \ldots, s_n) =_{\text{def}} (s'_{i1}, \ldots, s'_{in})$ such that

$$s'_{ij} = \begin{cases} s_{ij} + \Delta_{i,p_i} & \text{if } j = p_i \\ s_{ij} - \Delta_{i,p_i} \frac{s_{ij}}{t_i - s_{i,p_i}} & \text{if } j \neq p_i \end{cases}$$

Here, we assume $\frac{0}{0} =_{\text{def}} 1$.

(a) Design a Netlogo program to simulate the game Γ for 6 persons with available times $(3, 2, 2, 1, 1, 1)$ assuming that each person updates its time distribution according to the given local transition functions.

(b) Report the fixed point obtained in 10 runs of your Netlogo program assuming that the initial distributions are always the uniform distributions, i.e., each person $i \in A$ shares time $\frac{t_i}{n}$ with each person including herself.