UNIVERSITY OF KONSTANZ
DEPARTMENT OF COMPUTER & INFORMATION SCIENCE
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Network Dynamics
Winter 2013/14

Assignment 11

Ausgabe: 15 Jan 2014 **Abgabe:** 22 Jan 2014

Problem 1: Potential games — 10 Points

(a) Prove or disprove that the following bimatrix game

$$\Gamma_1 = \begin{pmatrix} (0,3) & (1,2) \\ (3,1) & (2,0) \end{pmatrix}$$

is a potential game.

Hint: Use the characterization mentioned in the lecture.

(b) Prove or disprove that the following bimatrix game

$$\Gamma_2 = \begin{pmatrix} (1,0) & (2,0) \\ (2,0) & (0,1) \end{pmatrix}$$

is an ordinal potential game.

Problem 2: Friendship networks — 10 Points

Consider the formation of a friendship network of n neurotic persons. A neurotic person wants to have many friends but wants these friends not to be friends among each other. We formulate this scenario as a strategic game $\Gamma = (A, S, u)$ such that

- $A = \{1, \dots, n\}$ is the set of persons,
- $S = S_1 \times \cdots \times S_n$ where $S_i = \mathcal{P}(\{\ (i,j) \mid j \in A \setminus \{i\}\ \})$, i.e., i's strategy is basically a set of selected persons; here, we consider friendship as a directed relationship which needs not necessarily be mutually confirmed,
- $u = (u_1, \dots, u_n)$ where $u_i(s_1, \dots, s_n)$ is the number of pairs $\{j, k\}$ such that $(i,j), (i,k) \in s_i$ but neither $(j,k) \in s_j$ nor $(k,j) \in s_k$.

Find a Nash equilibrium of Γ for n persons.

Hint: The Nash equilibrium is not unique.

Problem 3: Netlogo **10 Points**

Consider again the neurotic-network formation process in Problem 2. Assume that the utility functions are modified. That is, we consider a game $\Gamma = (A, S, u')$ where A and S are the same as above but u'_i is defined as

$$u'_i(s_1, \dots, s_n) =_{\text{def}} \|s_i\| - \|\{\ \{j,k\} \mid (i,j),(i,k) \in s_i \wedge ((j,k) \in s_j \vee (k,j) \in s_k)\ \}\|,$$

i.e., i's utility is the out-degree minus the number of pairs of simply connected friends.

For the local transition functions, we further assume that each person i in response to a given strategy profile $s = (s_1, \dots, s_n)$ selects some strategy $\bar{s}_i \in S_i$ such that $u'_i(\bar{s}_i, s_i)$ is maximized.

(a) Design a Netlogo program to simulate the game Γ for 10 persons assuming that each person updates according to the given local transition functions.

(b) Chart the time-series of the average out-degree for 10 persons and 100 iterations, averaged over 10 runs.