University of Konstanz
Department of Computer & Information Science
PD Dr. Sven Kosub / Mehwish Nasim

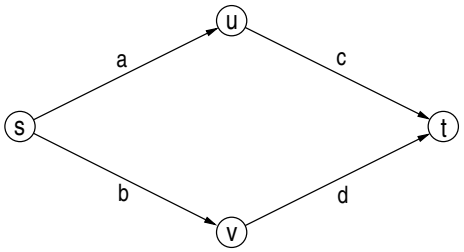
Network Dynamics
Winter 2013/14

Assignment 13

Ausgabe: 29 Jan 2014 **Abgabe:** 05 Feb 2014

Problem 1: Congestion games — 10 Points

Consider a congestion model $(A, F, (S_i)_{i\in A}, (w_f)_{f\in F})$ associated with the following traffic scenario:



where

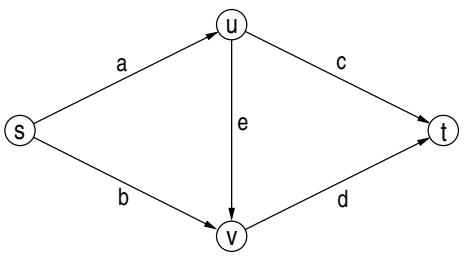
- the set A of agents is $A = \{1, 2, \ldots, 10\}$,
- the set F of facilities (roads) is $F = \{a, b, c, d\}$,
- the sets $S_i = S$ of strategies (pathways) is the same for all agents: $S = \{\ \{a, c\}, \{b, d\}\ \}$
- the cost functions w_f for each facility $f \in F$ are:

$$w_a(x) = x, \quad w_b(x) = 10, \quad w_c(x) = 10, \quad w_d(x) = x$$

Find a Nash equilibrium for the congestion game associated with the model above.

Problem 2: Braess paradox — 10 Points

Consider the congestion model $(A, F, (S_i)_{i\in A}, (w_f)_{f\in F})$, similar to one above, for a slightly modified traffic scenario:



where

- the set A of agents is $A = \{1, 2, \ldots, 10\}$,
- the set F of facilities (roads) is $F = \{a, b, c, d, e\}$,
- the sets $S_i = S$ of strategies (pathways) is the same for all agents:

$$S = \{\ \{a, c\}, \{b, d\}, \{a, e, d\}\ \}$$

- the cost functions w_f for each facility $f \in F$ are:

$$w_a(x) = x, \quad w_b(x) = 10, \quad w_c(x) = 10, \quad w_d(x) = x, \quad w_e(x) = 0$$

Find a Nash equilibrium for the congestion game associated with the model above.

(Why is it a paradox? *Hint*: Compare both traffic scenarios in Problem 1 and Problem 2.)

Problem 3: ROSENTHAL **potential** **10 Points**

Let a finite congestion model $(A, F, (S_i)_{i\in A}, (w_f)_{f\in F})$ be given. For the associated congestion game $\Gamma = (A, S, u)$ with $u_i(s) = \sum_{f\in s_i} w_f(\sigma_f(s))$, the ROSENTHAL *potential* P is defined for each $s \in S$ as

$$P(s) =_{\text{def}} \sum_{f\in\bigcup_{i\in A} s_i} \sum_{k=1}^{\sigma_f(s)} w_f(k)$$

Show that P is a potential function for the game Γ.