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Network Dynamics
WS 2010/11

Assignment 1

Ausgabe: 28 Oct 2010 **Abgabe:** 10 Nov 2010

Problem 1: System States — 10 Points

Consider the boolean system $S = (X, E, R)$ defined by population $X = \{0, 1, 2, 3\}$ of type $D = \{0, 1\}$, structure $E = C_4$ (an undirected cycle of length 4), and constraint R given by the following set of equations:

$$\begin{aligned} x_0 &= (x_3 \wedge x_1) \vee (x_3 \wedge \neg x_1) \vee (\neg x_3 \wedge x_1) \\ x_1 &= (x_0 \wedge x_2) \vee (x_0 \wedge \neg x_2) \vee (\neg x_0 \wedge x_2) \\ x_2 &= (x_1 \wedge x_3) \vee (x_1 \wedge \neg x_3) \vee (\neg x_1 \wedge x_3) \\ x_3 &= (x_2 \wedge x_0) \vee (x_2 \wedge \neg x_0) \vee (\neg x_2 \wedge x_0) \end{aligned}$$

Here, $\wedge$ denotes the logical AND, $\vee$ denotes the logical OR, and $\neg$ denotes the logical negation.

Find all states of the system S.

Problem 2: Constraints — 10 Points

You are given the following boolean system

$$S = (\{0, 1, 2, 3\}, P_3, \{(0, 1, 0, 1), (0, 1, 1, 1), (1, 1, 0, 1), (1, 1, 1, 1)\}).$$

Here, P_3 denotes an undirected path of length 3, i.e., the system's structure consists of the three undirected edges $\{0, 1\}, \{1, 2\}$, and $\{2, 3\}$. Suppose you want to explain the states of the system by a set of equations:

$$\begin{aligned} x_0 &= f_0(x_0, x_1) \\ x_1 &= f_1(x_0, x_1, x_2) \\ x_2 &= f_2(x_1, x_2, x_3) \\ x_3 &= f_3(x_2, x_3) \end{aligned}$$

Notice that set of equations corresponds to the interdependences given by P_3 (we omit loops).

Find functions f_0, f_1, f_2, and f_3 such that the set of solutions of the equation system above is exactly the constraint of the system S.

Note: You are only allowed to use $\wedge$, $\vee$, and $\neg$ to describe your functions.

Problem 3: Global State Dynamic **10 Points**

Consider a state process $P = (S_1, \dots S_h)$ such that for each $i \in [h]$, the system S_i consists of population $X = \{0, 1, 2\}$ of type $D = \{0, 1\}$, structure $E = K^3$, and constraint $R_i = D^3$.

Let $\mathbf{F}$ be the global state dynamic defined by

$$\mathbf{F} : D^3 \times [h-1] \to D^3 : (x_0, x_1, x_2) \mapsto (x_2 \oplus x_0 \oplus 1, x_0 \oplus x_1 \oplus 1, x_1 \oplus x_2 \oplus 1)$$

Obviously, $\mathbf{F}$ is compatible with P for each choice of h.

(a) Describe all trajectories generated by the global state dynamic for $h = \infty$.

(b) What is the smallest h such that the generated set of trajectories contains the complete information on the behavior of the trajectories?