

## Assignment 4

**Ausgabe:** 12 Jan 2011    **Abgabe:** 18 Jan 2011

### Problem 1: Functional Equivalence

**10 Points**

Let  $G = (X, E)$  be an arbitrary undirected graph. Show that there exists a family  $L$  of boolean local transition functions with interdependence structure  $E$  such that

$$\|\{ \mathbf{F}_L[\pi] \mid \pi \in S_X \}\| = \|\text{Acyc}(G)\|,$$

i.e., the number of different phase spaces of the local state dynamics  $(L, \pi)$  for permutations  $\pi$  is exactly the number of acyclic orientations of  $G$ .

### Problem 2: Chromatic Polynomial

**10 Points**

- (a) Determine the chromatic polynomial  $P_{\text{Path}_n}(x)$  of paths  $\text{Path}_n$  with  $n$  vertices.
- (b) Determine the chromatic polynomial  $P_{\text{Circ}_n}(x)$  of circuits  $\text{Circ}_n$  with  $n$  vertices.

*Hint:* Use induction.

### Problem 3: Chromatic Polynomial

**10 Points**

Show that the chromatic polynomial  $P_G(x)$  of an undirected graph  $G$  having  $n$  vertices and  $m$  edges is a polynomial of degree  $n$  such that the coefficient of  $x^n$  is 1 and the coefficient of  $x^{n-1}$  is  $-m$ .

*Hint:* Use induction.