

Assignment 2

Issue date: 01 May 2014 **Due date:** 08 May 2014, 11:00

It is explicitly recommended to solve exercises in groups of two.

Exercise 1: Counting graphs

2+2 Points

- Give the number of isomorphic graphs in each non-isomorphic graph class on 4 vertices.
- Draw one graph from each non-isomorphic graph class on 4 vertices.

Exercise 2: Degree sequences

2+2+4+2+2 Points

- (a) Prove that the number of vertices with odd degree is even in every simple graph $G = (V, E)$.
- (b) Prove:

$$\forall G = (V, E), \quad \exists u, v \in V : \deg(u) = \deg(v).$$

where $G = (V, E)$ is a simple graph with $|V| = n \geq 2$.

- (c) A sequence of natural numbers is called **graphic** if it is the degree sequence of a simple graph.

Which of the following degree sequences are graphic and which are not? Draw one example of a simple graph for each graphical sequence or explain why any of the sequence is not graphical.

- [6, 4, 3, 2, 2, 2, 2]
- [7, 5, 5, 3, 2, 2]
- [6, 5, 5, 2, 2, 2, 2]
- [6, 4, 4, 2, 2, 2, 2]
- [17, 15, 14, 12, 12, 10, 9, 8, 8, 7, 7, 6, 5, 5, 4, 3, 2, 2, 1, 1]

(d) Draw all the graphs that realize the following degree sequences:

- $[10, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$
- $[4, 4, 4, 4, 4]$
- $[2, 2, 2, 2, 2, 2, 2, 1, 1]$

If there are too many realizable graphs for any of the above degree sequences, it suffices to give an informal description instead of drawing all the possible graphs.

(e) A k -regular graph is a graph in which each vertex has degree k .
Draw 2 non-isomorphic 2-regular graphs with 6 vertices.

Please submit your answers electronically to teaching assistant Habiba (habiba@uni-konstanz.de).