UNIVERSITY OF KONSTANZ
ALGORITHMICS
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Assignment 3

Issue date: 08 May 2014 **Due date:** 15 May 2014, 11:00
It is explicitly recommended to solve exercises in groups of two.

Exercise 1: Degree sequences — 2+2 Points

(a) If two graphs have the same degree sequence, are they always isomorphic? If so, prove it. If not, give a counterexample.

(b) If two graphs are isomorphic, do they always have the same degree sequence? If so, prove it. If not, give a counterexample.

Exercise 2: Split graphs — 2+2+2+2+2+2+2 Points

(a) Argue that: "Preferential attachment graphs have low splittance".

(b) Prove:

$$\mathit{splittance}(P_n) = \begin{cases} n-4 & \textbf{if } n > 4 \\ 0 & \textbf{if } n \leq 4 \end{cases}$$

where P_n is a simple path of length $n-1$ on n vertices.

(c) Prove:

$$\mathit{splittance}(C_n) = \ n-3 \qquad \textbf{if } n \geq 3$$

where C_n is a cycle of length n on n vertices.

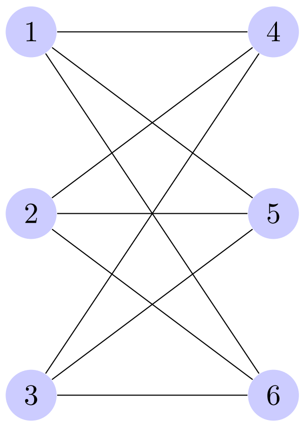
(d) Find all regular graphs that are split graphs.

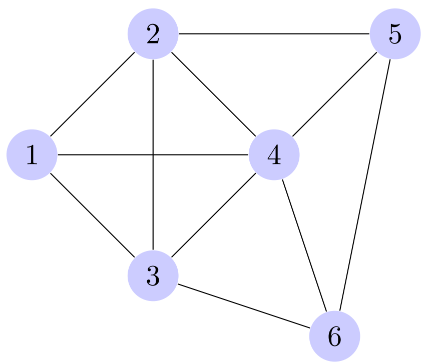
(*Hints: What does a degree sequence of a regular graph look like?*
The $k-th$ Erdős–Gallai equation for a split graph.)

(e) *G is a* **complete split graph**, *if each vertex in the independent set is adjacent to every vertex of the clique C.*
The **chromatic number of a graph** *G is the smallest number of colors needed to color the nodes of a graph G, such that, no two adjacent vertices share the same color.*
What is the chromatic number of a complete split graph with n vertices and clique of size c?

(f) Prove or disprove: "A complete split graph does not contain a P_4 as an induced subgraph".

(g) Calculate splittance of the following graphs:





Please submit your answers electronically to teaching assistant David (david.schoch@uni-konstanz.de).