University of Konstanz
Algorithmics
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Assignment 8

Issue date: 12 June 2014 **Due date:** 19 June 2014, 11:00
It is explicitly recommended to solve exercises in groups of two.

Exercise 1: Centralities 2 +2 + 2 + 2 Points

In the lectures on Centrality, a number of centrality measures were introduced that captured the structural importance of vertices with respect to their influence and importance. Here is an example of a slightly different kind of influence:
In a *Peer to Peer* mobile communication network, a relationship between a pair of nodes is defined as the message sent from one node to the other. The weight on such a relationship can be defined as the cost of sending the messages. In such a network it is preferred that each actor is connected through as few intermediaries as possible.

(a) Suggest a *graph centrality measure* that can capture the robustness of such a peer to peer network.

(b) Which centrality measure already introduced in the class is comparable to this measure? How?

(c) Give a formulation for measuring the centrality of individual actors based on this graph level measure.

(d) Is this measure different from betweenness centrality? If so, how? Moreover, can you propose a modification to betweenness centrality that capture the importance of vertices that would accommodate the above defined condition?

[please turn over]

Exercise 2: Structural Equivalence **5+5 Points**

Using the following definition from the lecture:

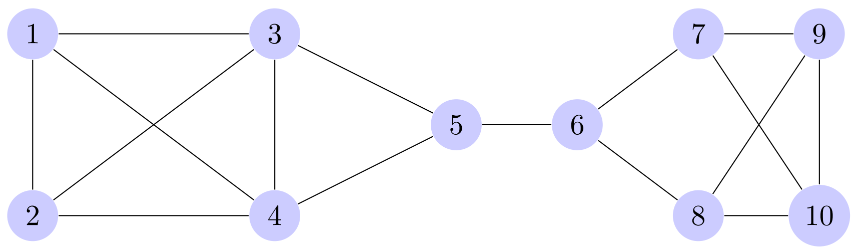
Let $G = (V, E)$ be a simple undirected graph. An equivalence relation $\sim \subseteq V \times V$ is called structural equivalence iff $\forall v, w \in V$:

$$v \sim w \Longrightarrow N(v) \setminus \{w\} = N(w) \setminus \{v\}.$$

(a) List all the partitions of the vertices of the following graph that correspond to the structurally equivalent classes of vertices.

(b) Order all the partitions in the order of finest refinement to coarsest refinement of the structurally equivalent classes.

Recall: A partition α of a set of vertices V is a refinement of a partition ρ of V or, we say that α is finer than ρ and that ρ is coarser than α, if every element of α is a subset of some element of ρ. Informally, this means that α is a further fragmentation of ρ.



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Exercise 3: Lattices **12 Points**

A *finite* poset $(P, \leq)$ is a lattice iff for all $S \in P$, $inf\{S\}$, $sup\{S\}$ exists in P.

Recall:

- $inf\{S\} \in P$, is the greatest element of the poset P that is less than or equal to all elements of the partition S of P.
- Dually, $sup\{S\} \in P$ is the least element of P that is greater than or equal to all elements of the partition S of P.
- Moreover, an upper semilattice is a poset that has a least upper bound for any non–empty finite subset and lower semilattice is a poset which has a greatest lower bound for any non–empty finite subset.

Prove that the following properties of a finite poset $(P, \leq)$ are equivalent.

(a) $(P, \leq)$ is a lattice.

(b) For all $S \in P$, $inf(S)$, $sup(S)$ exist in P.

(c) $(P, \leq)$ is an upper semilattice with bottom element.

(d) $(P, \leq)$ is a lower semilattice with top element.

Please submit your answers electronically to teaching assistant Habiba (habiba@uni-konstanz.de).