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Design and Analysis of Algorithms
WS 2012/2013

Assignment 3

Post Date: 07 Nov 2012 **Due Date:** 14 Nov 2012, 14:30
You are permitted and encouraged to work in groups of two.

Problem 1: Selection — 6 Points

Does the algorithm Select still work in linear time if the input elements are divided into groups of 3 and 7, respectively? Prove your statements.

Problem 2: Binary Counter — 7 Points

An array A of length k is used to implement a k-bit binary counter that counts upwards from zero. A binary number stored in the counter has its lowest-ordered bit in $A[0]$ and its highest-order bit in $A[k-1]$. The counter supports the operation Reset(A) that sets all bits to 0 and the operation Increment(A) implemented as follows:

Algorithm 1 Increment(A)

```
i ← 0
while i < length[A] and A[i] = 1 do
  A[i] ← 0
  i ← i + 1
end while
if i < length[A] then
  A[i] ← 1
end if
```

Give an upper bound for the running time of one Reset and n Increment operations

(a) based on the worst-case running time for each individual operation

(b) based on an amortized analysis of the $n + 1$ operations.

[please turn over]

Problem 3: **7 Points**

Consider a heap realized as an array A with operations INSERT and DELETEMIN. The worst-case costs of both operations are in $\mathcal{O}(\log n)$ where n is the number of elements in the heap.

(a) Give a potential function $\mathcal{C}$ such that the amortized costs of an INSERT-operation with respect to $\mathcal{C}$ are in $\mathcal{O}(\log n)$ and the amortized costs of a DELETEMIN-operation with respect to $\mathcal{C}$ are in $\mathcal{O}(1)$. Assume that the heap is empty at the beginning.

(b) The length of A can be dynamically adapted to the number of elements in the heap:

- Let the current length of A be $2^k - 1$ for a $k \in \mathbb{N}$.
- If A is completely filled, all elements in A are moved to a new array of length $2^{k+1} - 1$, before the 2^kth element is added.
- If A contains only 2^{k-2} elements, all elements in A are moved to a new array of length $2^{k-1} - 1$, after the 2^{k-2}th element is deleted.

Start with an empty heap and assume that the costs to move an element from one array to another are in $\mathcal{O}(1)$. What are the amortized costs of m INSERT-and DELETEMIN-operations?