

# How to compute Boyer-Moore shifts

## The good-suffix rule

For string  $s$  we define the set  $R(s)$  of all boundaries as

$$R(s) =_{\text{def}} \{s' \mid s' \text{ is a boundary of } s\}.$$

Now, the conditions that an admissible shift  $\sigma$  has to satisfy can be expressed as follows:

$$\sigma \leq j \wedge s_{j+1} \dots s_{m-1} \in R(s_{j+1-\sigma} \dots s_{m-1}) \wedge s_j \neq s_{j-\sigma} \quad (1)$$

$$\sigma > j \wedge s_0 \dots s_{m-1-\sigma} \in R(s_0 \dots s_{m-1}) \quad (2)$$

Define the array  $S$  containing the shortest admissible shift for each  $0 \leq j \leq m$  as

$$S[j] =_{\text{def}} \min \{ \sigma \mid (\sigma, j) \text{ fulfills condition (1) or fulfills condition (2)} \}.$$

This rule for computing  $S$  is called *good-suffix rule*. The computation of  $S$  according to the good-suffix rule can be done as described in the following algorithm:

Algorithm: COMPUTESHIFTS

Input: string  $s$  with  $|s| = m$

Output: array  $S$  containing shortest admissible shifts (according to the good-suffix rule)

1. FOR  $i := 0$  TO  $m$

2.      $S[i] := m$

      /\* computing shifts according to condition (1) \*/

3.      $H[0] := -1$

4.      $H[1] := 0$

5.     FOR  $j := 2$  TO  $m$

6.         WHILE  $k \geq 0$  AND  $s_{m-k-1} \neq s_{m-j}$

7.              $\sigma := j - k - 1$

8.              $S[m - k - 1] := \min\{S[m - k - 1], \sigma\}$

9.              $k := H[k]$

10.          $H[j] := k + 1$

11.          $k := k + 1$

      /\* computing shifts according to condition (2) \*/

12.      $B := \text{COMPUTEBOUNDARIES}(s)$

      /\* from KNUTH-MORRIS-PRATT algorithm \*/

13.      $j := 0$

14.      $i := B[m]$

15.     WHILE  $i \geq 0$

16.         WHILE  $j < m - i$

17.              $S[j] := \min\{S[j], m - i\}$

18.              $j := j + 1$

19.          $i := B[i]$

20.          $j := 0$

### The bad-character rule

How can we beneficially integrate the motivating idea that has led to the right-to-left approach? We define another rule called *bad-character rule*. Consider a mismatch at  $(i, j)$  caused by symbol  $t_{i+j} = x$ . There are two possible cases:

1. There is an  $0 \leq r \leq j - 1$  such that  $s_r = x$ . Then, define  $\sigma =_{\text{def}} j - r$ .
2. For all  $0 \leq r \leq j - 1$  it holds  $s_r \neq x$ . Then, define  $\sigma =_{\text{def}} j + 1$ .

It is easily seen that we can combine the *good suffix rule* and the *bad character rule* by taking the maximum of the shifts for each  $j$ .