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Network Dynamics
Winter 2015/16

Assignment 2

Ausgabe: 28 Oct 2015 **Abgabe:** 04 Nov 2015

Problem 1: Whole networks

(a) Let $A = \{1, 2, \ldots, 10\}$ be the set of items and let $\mathcal{I} = A \times A \setminus \{ \ (i, i) \mid i \in A \ \}$ be the interaction domain on A. Suppose you are given positive real numbers $w_1, w_2, \ldots, w_{10}$ and θ. Consider the following (single-attribute) network

$$x \ : \ \mathcal{I} \to \mathbb{R} \ : \ (i, j) \ \mapsto \ \max\{w_i + w_j - \theta, 0\}.$$

Visualize the weighted, undirected graph of the (single-attribute) network x for $w_i = 1/i$ and $\theta = 3/10$.

(b) Consider again the network x as in subproblem (a) for the same values for w_i. Visualize the weighted, undirected graph of x for $\theta = 1/6$.

Problem 2: Two-mode networks

Let $A = \{1, \ldots, n\}$ be a set of (enumerated) politicians and let $S = \{1, \ldots, m\}$ be a set of (enumerated) boards convening at distinct times. Suppose politician i attends a meeting of board k with probability p_{ik}. We are interested in the corresponding board communication network. More specifically, we are interested in the following network

$$x \ : \ \mathcal{I} \to [0, 1] \ : \ (i, j) \mapsto \text{probability that politicians } i \text{ and } j \text{ meet in some board meeting}$$

where $\mathcal{I} = A \times A \setminus \{ \ (i, i) \mid i \in A \ \}$ is the interaction domain on A.

Find an appropriate representation of the network x for $n = 5$, $m = 3$, and the following probability matrix:

$$\begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/3 & 0 & 2/3 \\ 1/4 & 0 & 0 \\ 1/2 & 1/10 & 3/10 \\ 0 & 1 & 0 \end{pmatrix}$$

Problem 3: Two-mode networks

Suppose you are given a bipartite graph $G = (V, E)$ such that $V = V_1 \cup V_2$ for two disjoint sets V_1 and V_2. For $i \in \{1, 2\}$, let $\Delta_i(G)$ denote the maximum degree of a vertex $v \in V_i$. Assume that $G = G(x)$ represents a two-mode network x (on an affiliation domain $V_1 \times V_2$). For $i \in \{1, 2\}$, let G_i be the graph of the one-mode projection of x on V_i.

(a) Find a possibly tight upper bound on the value $\max\{\Delta(G_1), \Delta(G_2)\}$ depending on $\Delta_1(G)$ and $\Delta_2(G)$.

(b) Find a possibly tight lower bound on the value $\max\{\Delta(G_1), \Delta(G_2)\}$ depending on $\Delta_1(G)$ and $\Delta_2(G)$.