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Network Dynamics
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Assignment 6

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Problem 1: Lions and lambs

We consider a 2-person game where the two agents have a lion strategy (i.e., an aggressive strategy) and a lamb strategy (i.e., a peaceful strategy). The goal of the game is to gain possession of a valuable good.

If both agents choose the lion strategy, they will struggle for the prey. If both agents choose the lamb strategy, they will share the prey. If agents choose different strategies, the agent with the lamb strategy delivers the prey to the other agent. More specifically,

- V is the profit of gaining exclusive possession of the prey,
- $D/2$ is the loss for each agent when struggling for the prey, and
- $W/2$ is the additional profit for each agent when sharing the prey without struggle.

Formulate the game as bimatrix game and find Nash equilibria (depending on V, D, W).

Problem 2: Friendship network

Consider the friendship network from the lecture, which is formed by the following game $\Gamma = (A, S, u)$:

- $A = \{1, \ldots n\}$ where each agent has an amount $t_i \geq 0$ of spare time available to friends
- $S = S_1 \times \cdots \times S_n$ where

  $$S_i =_{\text{def}} \{\ (s_{i1}, \ldots, s_{in}) \mid s_{ij} \geq 0 \text{ and } s_{i1} + \cdots + s_{in} = t_i\ \}$$

  for each $i \in A$.
- $u = (u_1, \ldots, u_n)$ where

  $$u_i(s_1, \ldots, s_n) =_{\text{def}} \sum_{\substack{j=1 \\ i \neq j}}^{n} \min\{s_{ij}, s_{ji}\}$$

  for each $i \in A$ and each strategy profile s

We say that a strategy profile $s = (s_1, \ldots, s_n)$ is *efficient* if and only if $s_{ii} = 0$ for all $i \in A$.

(a) Is there an efficient Nash equilibrium for a group of four friends with $t_1 = 70, t_2 = 50, t_3 = 100$, and $t_4 = 40$?

(b) Find a necessary criterion for the existence of an efficient Nash equilibrium for the game with n agents and times $(t_1, \ldots, t_n)$.

Problem 3: Co-authorship networks

In the co-author model, each agent is a researcher who spends time working on research projects. If two researchers are connected in the network then they are working on a project together. Each agent has a fixed amount of time to spend on research, and so the time that researcher i spends on a given project is inversely related to the number of projects, p_i, that she is involved in. The synergy between two researchers i and j depends on how much time they spend together, and this is captured by a term $\frac{1}{p_i p_j}$. Here, the more projects a researcher is involved in, the lower the synergy that is obtained per project.

In the following, assume that k projects are available to n researchers. Each project may involve an arbitrary number of researchers. We formalize this scenario as game $\Gamma = (A, S, u)$:

- $A = \{1, \ldots, n\}$
- $S = S_1 \times \cdots \times S_n$ where $S_i = \{ \ P \mid \emptyset \neq P \subseteq \{1, \ldots, k\} \ \}$ for each $i \in A$, i.e., researcher i chooses some set of projects (but at least one project) she wants to be involved in
- $u = (u_1, \ldots, u_n)$ where

$$u_i(P_1, \ldots, P_n) =_{\text{def}} \sum_{P_i \cap P_j \neq \emptyset} \left(\frac{1}{\|P_i\|} + \frac{1}{\|P_j\|} + \frac{1}{\|P_i\| \cdot \|P_j\|} \right)$$

  for each $i \in A$ and all strategy profiles. Note that in the sum the indexes i, j need not be distinct.

Find a Nash equilibrium for Γ and show the co-authorship network (depending on n, k).