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Network Dynamics
Winter 2015/16

Assignment 7

Ausgabe: 09 Dec 2015 **Abgabe:** 16 Dec 2015

Problem 1: Stable networks

Consider the connections model for five agents, $0 < \delta < 1$, and $c > 0$.

Find a necessary and sufficient criterion for δ and c such that the circular graph C_5 is stable.

Problem 2: Efficient networks

Consider the connections model for the set $A = \{1, \dots, n\}$ of agents, $0 < \delta < 1$, and $c > 0$.

A graph $G = (A, E)$ is said to be *efficient* if and only if it maximizes the summation of each agent's utility, i.e.,

$$U(G) =_{\text{def}} \sum_{i \in A} u_i(G) \geq \sum_{i \in A} u_i(G') = U(G')$$

for all graphs $G' = (A, E')$.

Prove the following statements:

(a) If $\delta - c > \delta^2$ then the complete graph K^n is efficient.

(b) If $\delta - c < \delta^2$ and $c < \delta + \frac{n-2}{2} \cdot \delta^2$ then a star graph $K_{1,n-1}$ is efficient.

(c) If $\delta - c < \delta^2$ and $c \geq \delta + \frac{n-2}{2} \cdot \delta^2$ the the empty graph is efficient.

(d) Efficient graphs are unique.

Problem 3: Convergence to networks

Consider the connections model for four agents, $0 < \delta < 1$, and $c > 0$ such that $0 < \delta - c < \delta^2$, i.e., a star graph $K_{1,3}$ is stable. Furthermore, consider the myopic network formation process from the lecture.

Find sequences of dyads such that the myopic process converges to non-isomorphic graphs.