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Department of Computer & Information Science
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Network Dynamics
Winter 2015/16

Assignment 9

Ausgabe: 13 Jan 2016 **Abgabe:** 20 Jan 2016

Problem 1: Ordinal potential games

Prove or disprove the following statement:

> Let $\Gamma = (A, S, u)$ be a game with utilities, $\|A\| = 2$, $\|S_1\| = \|S_2\| = 2$. Then, Γ is an ordinal potential game if and only if Γ has a Nash equilibrium.

Problem 2: Potential games

In the following, apply the characterization of potential games by closed paths, as given in the lecture.

(a) Prove or disprove that the following bimatrix game

$$\Gamma = \begin{pmatrix} (1,3) & (2,3) & (3,3) \\ (2,2) & (3,1) & (1,3) \\ (3,3) & (1,1) & (2,2) \end{pmatrix}$$

is a potential game.

(b) Suppose you are given an $n \times k$-game $\Gamma = (A, S, u)$, i.e., a game with n agents each of which has k strategies at hand. Depending on n and k, how many closed paths do you have to check in order to decide whether Γ is a potential game.

Problem 3: Friendship network

Consider the friendship network from the lecture, which is formed by the following game $\Gamma = (A, S, u)$:

- $A = \{1, \dots n\}$ where each agent has an amount $t_i \geq 0$ of spare time available to friends
- $S = S_1 \times \cdots \times S_n$ where

$$S_i =_{\text{def}} \{\ (s_{i1}, \dots, s_{in}) \mid s_{ij} \geq 0 \text{ and } s_{i1} + \cdots + s_{in} = t_i\ \}$$

for each $i \in A$.

- $u = (u_1, \ldots, u_n)$ where

$$u_i(s_1, \ldots, s_n) =_{\text{def}} \sum_{\substack{j=1 \\ i \neq j}}^{n} \min\{s_{ij}, s_{ji}\}$$

  for each $i \in A$ and each strategy profile s

Prove or disprove that the game $\Gamma = (A, S, u)$ is a potential game.