University of Konstanz
Department of Computer & Information Science
PD Dr. Sven Kosub

Network Dynamics
WS 2011/12

Assignment 1

Ausgabe: 2 Nov 2011 **Abgabe:** 9 Nov 2010

Problem 1: Congestion model — 10 Points

Suppose you are given the congestion model $(\{a,b\},\{\{a\},\{b\}\},(c_a,c_b))$ with cost functions $c_a(x) = 1$ and $c_b(x) = x^r$ for $r > 1$, i.e., a traffic network connecting a starting and a destination via two parallel pathways $\{a\}$ and $\{b\}$ each consisting of a single road a or b.

(a) A user distribution x is said to be at *Wardrop equilibrium* if $c_s(x) \leq c_{s'}(x)$ for all $s, s' \in S$ such that $x(s) > 0$. Determine a user distribution at Wardrop equilibrium for the above congestion model. What is the social cost of the user distribution?

(b) A user distribution x^* is said to be a *social optimum* if $C(x^*) \leq C(x)$ for all user distributions x. Determine a social optimum. What is the social cost of the social optimum?

(c) Let x^* be a social optimum and let x be a user distribution at Wardrop equilibrium. The *coordination ratio* γ^* is the infimum of all γ such that $C(x) \leq \gamma \cdot C(x^*)$. Determine the coordination ratio for the above congestion model.

Problem 2: Phase space — 10 Points

Consider the following simulation: We have an interdependence graph $G = (V, E)$ consisting of vertex set $V =_{\text{def}} \{0,1,2,3\}$ and edge set $E =_{\text{def}} K_{1,3} = \{\ \{0,1\},\{0,2\},\{0,3\}\ \}$. Suppose that there is a function f_i assigned to each vertex i as follows:

$$\begin{aligned} f_0(x_0,x_1,x_2,x_3) &=_{\text{def}} x_0 \oplus x_1 \oplus x_2 \oplus x_3, \\ f_1(x_0,x_1) &=_{\text{def}} x_0 \oplus x_1, \\ f_2(x_0,x_2) &=_{\text{def}} x_0 \oplus x_2, \\ f_3(x_0,x_3) &=_{\text{def}} x_0 \oplus x_3, \end{aligned}$$

where $\oplus$ denotes addition modulo 2.

Describe the phase space for the sequential update schedule $\pi = (1,2,3,0)$.

Problem 3: Update order — 10 Points

We consider sequential simulation behaviors on a circuit Circ_n of size n. Prove that there are at most $2^n - 2$ different phase spaces.

Hint: Find an appropriate recursive formula and use induction.