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Network Dynamics
Winter 2015/16

Assignment 10

Ausgabe: 20 Jan 2016 **Abgabe:** 27 Jan 2016

A *congestion model* is a tuple $(A, F, (S_i)_{i\in A}, (w_f)_{f\in F})$ such that

1. $A = \{1, \ldots, n\}$ is a non-empty, finite set of *agents*,
2. F is a non-empty, finite set of *facilities*,
3. $S_i \subseteq \mathcal{P}(F)$ is a non-empty set of strategies for each agent $i \in A$, and
4. $w_f : \{1, \ldots, n\} \to \mathbb{R}$ is a *cost function* for each facility $f \in F$; if k agents choose f then the cost for each agent is $w_f(k)$.

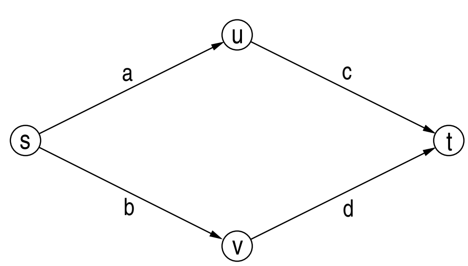
Let $(A, F, (S_i)_{i\in A}, (w_f)_{f\in F})$ be a congestion model. Then, $\Gamma = (A, (S_i)_{i\in A}, u)$ is called *congestion game* if and only if for all $i \in A$, $s = (s_i, s_{-i}) \in S$,

$$u_i(s) = \sum_{f\in s_i} w_f(\sigma_f(s)),$$

where $\sigma_f(s) = \|\{i \in A | f \in s_i\}\|$.

Problem 1: Congestion games 10 Points

Consider a congestion model $(A, F, (S_i)_{i\in A}, (w_f)_{f\in F})$ associated with the following traffic scenario:



where

- the set A of agents is $A = \{1, 2, \ldots, 10\}$,
- the set F of facilities (roads) is $F = \{a, b, c, d\}$,

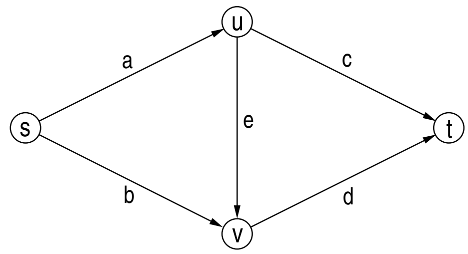
- the sets $S_i = S$ of strategies (pathways) is the same for all agents: $S = \{ \{a, c\}, \{b, d\} \}$
- the cost functions w_f for each facility $f \in F$ are:

$$w_a(x) = x, \quad w_b(x) = 10, \quad w_c(x) = 10, \quad w_d(x) = x$$

Find a Nash equilibrium for the congestion game associated with the model above.

Problem 2: BRAESS **paradox** **10 Points**

Consider the congestion model $(A, F, (S_i)_{i \in A}, (w_f)_{f \in F})$, similar to one above, for a slightly modified traffic scenario:



where

- the set A of agents is $A = \{1, 2, \ldots, 10\}$,
- the set F of facilities (roads) is $F = \{a, b, c, d, e\}$,
- the sets $S_i = S$ of strategies (pathways) is the same for all agents:

$$S = \{ \{a, c\}, \{b, d\}, \{a, e, d\} \}$$

- the cost functions w_f for each facility $f \in F$ are:

$$w_a(x) = x, \quad w_b(x) = 10, \quad w_c(x) = 10, \quad w_d(x) = x, \quad w_e(x) = 0$$

Find a Nash equilibrium for the congestion game associated with the model above.

(Why is it a paradox? *Hint*: Compare both traffic scenarios in Problem 1 and Problem 2.)

Problem 3: ROSENTHAL **potential** **10 Points**

Let a finite congestion model $(A, F, (S_i)_{i \in A}, (w_f)_{f \in F})$ be given. For the associated congestion game $\Gamma = (A, S, u)$ with $u_i(s) = \sum_{f \in s_i} w_f(\sigma_f(s))$, the ROSENTHAL *potential* P is defined for each $s \in S$ as

$$P(s) =_{\text{def}} \sum_{f \in \bigcup_{i \in A} s_i} \sum_{k=1}^{\sigma_f(s)} w_f(k)$$

Show that P is a potential function for the game Γ.