University of Konstanz
Department of Computer & Information Science
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Network Dynamics
Winter 2015/16

Assignment 3

Ausgabe: 04 Nov 2015 **Abgabe:** 11 Nov 2015

Problem 1: Open triangles

For a network $x : \mathcal{I} \to R$ $(= R^{\mathcal{I}})$, let $\mathbf{F} : R^{\mathcal{I}} \to R^{\mathcal{I}}$ be that map that closes all open triangles, i.e., induced paths P_3, in the graph $G(x)$ of (single-attribute) network x. We say that two paths are *edge-incident* if and only if there is an edge which belongs to both paths.

(a) Which connected graph on n vertices has the highest number of edge-incident induced P_3?

(b) Which connected graph on n vertices has the highest number of non-edge-incident induced P_3?

(c) Find a function $f : \mathbb{R}_{>0} \to \mathbb{R}_{>0}$ (possibly small) such that for all connected, non-complete networks x,

$$\operatorname{diam}(G(\mathbf{F}(x))) \leq f(\operatorname{diam}(G(x)).$$

A network x is connected (complete) iff the graph $G(x)$ of the network x is connected (complete).

Problem 2: Thresholds

An n-ary function $f : \{0,1\}^n \to \{0,1\}$ is a *threshold function* if and only if there are weights $w_1, \ldots, w_n \in \mathbb{R}_{\geq 0}$ and a threshold $\theta \in \mathbb{R}_{\geq 0}$ such that for all $z_1, \ldots, z_n \in \{0,1\}$,

$$f(z_1, \ldots, z_n) = 1 \iff \sum_{i=1}^{n} w_i \cdot z_i \geq \theta_i$$

If possible, find weights and thresholds such that the following boolean functions are threshold functions:

(a) $x_1 \wedge x_2$

(b) $x_1 \vee x_2$

(c) $(x_1 \wedge x_2) \vee x_3$

(d) $(x_1 \wedge x_2) \vee (x_3 \wedge x_4)$

Problem 3: Job rotation

Find a map $\mathbf{F}$ for the network y in the job rotation scenario given in the lecture (see also the lecture notes, version v5.2 or higher) without recurring to the trainee assignment π.