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Network Dynamics
Winter 2015/16

Assignment 5

Ausgabe: 18 Nov 2015 **Abgabe:** 01 Dec 2015

We consider psychology-of-conformity dynamics from the lecture, which is given by the following transition matrix P of a Markov chain with finite state space $J = \{0,1\}^3$:

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/3 & 1/2 & 0 & 1/6 & 0 & 0 & 0 & 0 \\ 1/3 & 0 & 0 & 1/3 & 0 & 0 & 1/3 & 0 \\ 0 & 1/6 & 0 & 1/2 & 0 & 0 & 0 & 1/3 \\ 1/3 & 0 & 0 & 0 & 1/2 & 0 & 1/6 & 0 \\ 0 & 1/3 & 0 & 0 & 1/3 & 0 & 0 & 1/3 \\ 0 & 0 & 0 & 0 & 1/6 & 0 & 1/2 & 1/3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Note that states are enumerated in lexicographical order, i.e., $000, 001, 010, \dots, 111$.

Problem 1: Transition probabilities

For the Markov chain above with state space $J = \{0,1\}^3$ and transition matrix P. Consider the initial distributions:

$$q^{(0)} = (1/8, \dots, 1/8), \qquad r^{(0)} = (1/2, 1/14, \dots, 1/14)$$

(a) Determine the distributions $q^{(1)}$ and $q^{(2)}$.

(b) Determine the distributions $r^{(1)}$ and $r^{(2)}$.

(c) Do the sequences $(q^{(t)})_{t \in \mathbb{N}}$ and $(r^{(t)})_{t \in \mathbb{N}}$ converge? If "yes," what are the limits of the sequences?

Hint: You may use appropriate software.

Problem 2: Hitting time

Let $(X_0, X_1, \dots)$ be a Markov chain with finite state space J and transition matrix P. The *hitting time* $T_{i,j}$ for state $j \in J$ of the Markov chain starting in state $i \in J$ (i.e., $X_0 = i$) is the random variable defined by

$$T_{i,j} =_{\text{def}} \min \{ \ t \in \mathbb{N}_+ \mid X_t = j \text{ if } X_0 = i \ \},$$

where $T_{i,j} =_{\text{def}} \infty$ if j is never reached from i. The *mean hitting time* $\tau_{i,j}$ is defined by

$$\tau_{i,j} =_{\text{def}} \mathbf{E}(T_{i,j}).$$

In the following, consider the Markov chain above with state space $J = \{0,1\}^3$.

(a) Determine the distribution of the hitting time $T_{001,000}$.

(b) Determine the mean hitting time $\tau_{001,000}$.

(c) Determine the distribution of the hitting time $T_{010,000}$.

(d) Determine the mean hitting time $\tau_{010,000}$.

Problem 3: Return time

Let $(X_0, X_1, \dots)$ be a Markov chain with finite state space J and transition matrix P. Then, hittin time $T_{i,i}$ is called *return time*, mean hitting time $\tau_{i,i}$ is called *mean return time*.

Determine the mean return times for all states $i \in \{0,1\}^3$ of the Markov chain above.

Hint: Use symmetries of the Markov chain and among the states.