University of Konstanz
Department of Computer & Information Science
Prof. Dr. Sven Kosub / Dr. David Schoch

Network Dynamics
Winter 2015/16

Assignment 8

Ausgabe: 16 Dec 2015 **Abgabe:** 13 Jan 2016

In the following, we consider game-theoretic models of strategic network formation with structural holes for n agents:

- $A = \{1, \ldots, n\}$ is the set of persons,
- $S = S_1 \times \cdots \times S_n$ where $S_i = \mathcal{P}(\{\ (i,j) \mid j \in A \setminus \{i\}\ \})$, i.e., i's strategy is basically a set of selected persons; here, we consider friendship as a directed relationship which needs not necessarily be mutually confirmed.

Utilities will reflect differences between the models.

Problem 1: Heterogeneous costs

Let $c_{i,j} \geq 0$ denote agent i's cost of buying a link to agent j. The utility function $u = (u_1, \ldots, u_n)$ is given as follows for each $i \in A$ (similar to the utility function discussed in the lecture):

$$u_i(s_1, \ldots, s_n) =_{\text{def}} \alpha_0 \cdot (\|s_i\| + \{\ j \mid (j,i) \in s_j\ \}\|) + \sum_{(i,j),(i,k) \in s_i} \beta(r_{j,k}) - \sum_{(i,j) \in s_i} c_{i,j},$$

where $\alpha_0 \geq 0$, β is a decreasing, non-negative function, and $r_{j,k}$ is the number of length-2 paths in the underlying undirected graph induced by the strategies of the agents.

Find a Nash equilibrium for the case of five agents, i.e., $A = \{1,2,3,4,5\}$, and cost model:

$$c_{i,j} =_{\text{def}} \begin{cases} j - i & \text{if } j \geq i \\ 5 & \text{if } j < i \end{cases}$$

Problem 2: Neurotic persons

Consider the formation of a friendship network of n neurotic persons. A neurotic person wants to have many friends but wants these friends not to be friends among each other. We formulate this scenario by utility function $u = (u_1, \ldots, u_n)$ where $u_i(s_1, \ldots, s_n)$ is the number of pairs $\{j,k\}$ such that $(i,j), (i,k) \in s_i$ but neither $(j,k) \in s_j$ nor $(k,j) \in s_k$.

Find (ideally, all) Nash equilibria of Γ for n persons.

Problem 3: Neurotic persons

Consider again the neurotic-network formation process in Problem 2. Assume that the utility function u_i is modified as follows for each $i \in A$:

$$u'_i(s_1, \ldots, s_n) =_{\text{def}} \|s_i\| - \|\{\ \{j,k\} \mid (i,j),(i,k) \in s_i \wedge ((j,k) \in s_j \vee (k,j) \in s_k)\ \}\|,$$

i.e., i's utility is the out-degree minus the number of pairs of simply connected friends.

Find (ideally, all) Nash equilibria of Γ for n persons.

Merry Christmas and a Happy New Year!